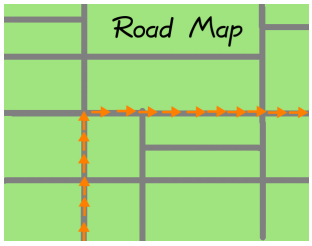
Module 1.2: Basic Venn Diagram Problems



The purpose of a Venn Diagram is usually to communicate information about a set in a visual form. They also have other uses. Venn Diagrams can help us calculate missing information about sets, and they can also help us test and understand set-theory expressions. If you've never heard of a Venn Diagram before, feel free to casually flip through this module and the next two, and observe all the colorful diagrams made up of circles. Eventually, we will use Venn Diagrams to solve complicated problems in combinatorics and probability.

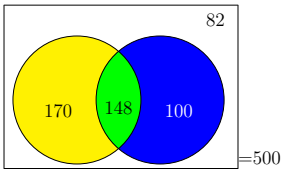
Basically, Venn Diagrams come in two forms: one form is for counting problems, and the other form is for determining what is in a set, and what is not. The latter category of problems are sometimes called "shading problems." In this module, we'll mostly focus on counting problems.



Suppose a marketing survey for satellite radio is sent to 500 subscribers and asks them if they listen to music stations on the way to work, and also if they listen to news stations on the way to work. Suppose that 318 people listen to music, and 248 people listen to news.

The problem with that is $318 + 248 = 566$ and there are only 500 people in our survey! What is wrong? Have you been deceived? Has there been a tabulation error in the survey? Actually, neither of those is the case. This type of error is very common, and is called "double counting." The following example will make the situation clear.

Now suppose the marketers go back to *the same collection of 500 people*, and discover that 82 do not listen to either music nor the news on the way to work. At first, this seems worse, because now we have $566 + 82 = 648$ people in our 500 person survey! However, after you read this module, you will be able to perform some calculations and construct the following diagram:



I'd like to take a moment to explain what that diagram means.

- The left circle, containing $170 + 148 = 318$ people, represents the 318 people who listen to music.
- The right circle, containing $148 + 100 = 248$ people, represents the 248 people who listen to news.
- As you can see, the 148 is inside both circles. That represents people who both listen to music and listen to news. We'll call this region "football shaped" in the rest of this chapter.
- The 170 represents people who listen to music but not the news (because it is inside the left circle and not inside the right circle).
- Similarly, the 100 represents people who listen to news but not music (because it is inside the right circle and not the left circle).
- Regions like those containing the 170 and the 100, which look like a circular cookie with a bite taken out of it, will be called "moon shaped."
- The 82 is outside both circles, and represents people who listen to neither music nor the news. We'll call this "the background."
- Finally, the $= 500$ tells us that the whole Venn Diagram contains $170 + 148 + 100 + 82 = 500$ people.

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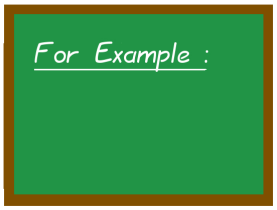
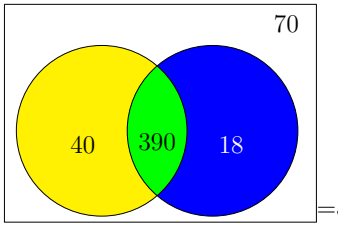


1-2-1

Suppose we are going to examine the feasibility of a credit-card marketing campaign on a campus. First we will classify students as to whether they have student loans or not, and whether or not they have a job. Out of 518 students surveyed, 430 have student loans, and 408 have jobs. In fact, 390 have both, and only 70 students have neither. The way to show this in a Venn Diagram is as follows



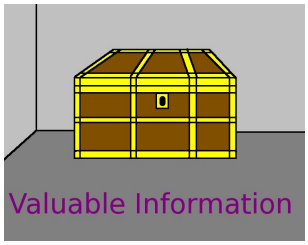
=518

where the left circle represents students with loans and the right circle represents students with jobs. We'll continue in the next box.

Continuing with the previous box, here are some steps that can lead to the above diagram. However, this is one of those situations, common in mathematics, where there are many paths to a solution.

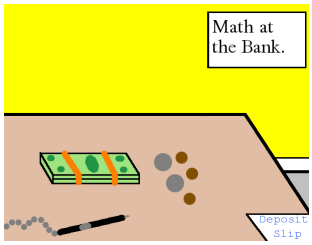
- Those in the background are those with neither a job nor a loan (given as 70).
- The moon-shaped part on the left are those with loans but no job (430-390=40.)
- Those on the right are those with jobs but no loans (408-390=18).
- The football shaped part in the middle is where people with both jobs and loans are placed (given as 390).
- Finally, the number outside the diagram is the total number of people counted in the diagram. This set is called the *universal set* or *universe* of the problem, its size can be a check on your work.

$$40 + 390 + 18 + 70 = 518$$



There is a sentence from the previous box that is very important. I'd like to take a moment to highlight it for you.

The set of people or objects counted in a Venn Diagram is called the *universal set* or *universe* of the problem. Among other things, it can help you check your work.



Throughout this chapter, we'll see problems relating to databases, a phenomenally important subject in today's data-driven business world. Many database companies work in the fields of health care or education. These are lucrative markets, particularly because hospitals and universities are well-funded. However, both of these are also highly-regulated markets in most countries, including in the USA.

Any health-related database of patients is governed (in the USA) by a regulation called HIPAA. Any education-related database is governed (in the USA) by a regulation called FERPA. Companies must take great care to comply with HIPAA or FERPA, as appropriate, to avoid heavy fines. I'm mentioning those now, so that you will not be surprised if you hear these acronyms in a job interview.

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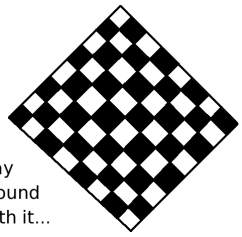
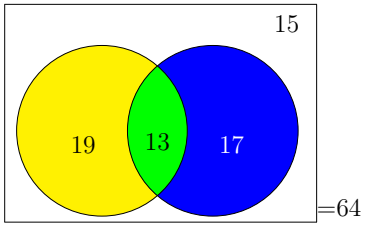


1-2-2

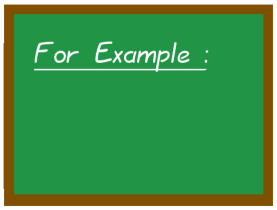
Suppose you work for a company that makes databases. They have to keep track of how many of their employees are trained in the regulations mentioned in the previous box. Consider the following Venn Diagram. The left circle represents the number of employees at a company who have received training in HIPAA, and the right circle represents employees who have received training in FERPA.



=64

- How many employees have training in FERPA?
- How many employees have training in HIPAA?
- How many employees have training in both?
- How many employees have training in either?
- How many employees have training in neither?

The answers will be given on Page 77. Please try to answer all five questions before looking up the answers.

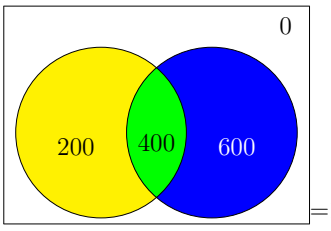


1-2-3

Suppose that every single freshman at a particular liberal-arts college has to take history or philosophy during their first semester. There are 400 people in both, and 1200 freshmen in the school. If 600 are taking philosophy, how many are taking history?

Well first, 1200 people form the problem's *universal set* or *universe*, so that goes with the equal sign outside of the Venn Diagram. Freshmen are forbidden from dodging both history and philosophy, so we put 0 in the background of the diagram. Clearly the 400 goes in the football-shaped region in the middle. Let the left circle be philosophy. If 600 people are in philosophy, and 400 people are in both, then clearly 200 are in philosophy alone. Then I have accounted for 600 out of 1200. The remaining 600 must be in the only remaining spot, those who are taking history alone. Finally, the number of students in history is therefore 1000, being the sum of 600 taking it alone and 400 taking it with philosophy. (The final diagram is given in the next box.)

Here is the Venn Diagram for the previous box.



=1200

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For Example :

1-2-4

Imagine that you are doing some data collection on the admissions department of your university, for the international commerce MBA program. As you might expect, that program requires students to be proficient in at least 2 languages. They also prefer, but do not require, students to have done a study abroad program or students who have proficiency in 3 or more languages. Of the 478 admitted applicants, you discover that 286 have proficiency in three or more languages; of those, 190 had done study-abroad programs. You also find that 84 admitted applicants have proficiency only in two languages, but have not done a study-abroad program. How many of the admitted students have proficiency in only two languages, but did a study-abroad program?

The solution is in the next box.

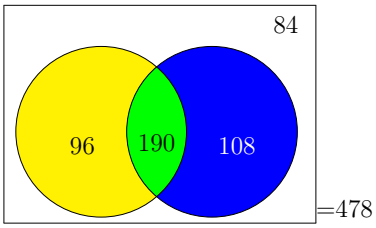
Now we'll solve the problem from the previous box.

While a Venn Diagram is not requested, it would be extremely useful. Let the left circle be the students who have three or more languages, and the right circle those who have studied abroad. Definitely, the 190 goes in the middle, in the football-shaped part. Then $286 - 190 = 96$ are students who have three or more languages, but who did not study abroad—they go into the left moon-shaped part. Next, the 84 people go into the background part (outside the circles) because they neither did a study abroad, nor do they have proficiency in three or more languages.

The right-hand moon-shaped part is the only part remaining, and that's also what we need to find, because the question asked "How many of the admitted students have proficiency in only two languages, but did a study-abroad program?" So we see that $478 - 84 - 96 - 190 = 108$ people should be placed there, and that is our final answer.

The Venn diagram is in the next box.

Here is the Venn Diagram for the previous example.



For Example :

1-2-5

Your friend is doing research on student employment on campus. He has surveyed 100 graduating seniors. Of these, 30 have never had an internship nor do they have a job lined up for after graduation. He finds that 68 have had internships, and of those, 48 have a job lined up. The rest of his data is unreadable because he spilled coffee on his notes. Draw a Venn Diagram to represent the data, and tell him how many people have a job lined up for after graduation, and how many of those never had an internship.

We start by noting 100 on the outside of the box, that's the size of the problem's universal set (or universe). Let the left circle represent students who've had internships, and the right circle represent students with a job lined up for after graduation. Then 30 have neither a job lined up nor have had an internship, so we put that in the background away from the other circles. Now, we have 70 people to allocate among the union of the two circles. Since 48 have both a job lined up and an internship in their past, we record that in the football-shaped part in the middle. We know that 68 people have had internships in total, so $68 - 48 = 20$ have had internships but do not have a job lined up. Then the 20 goes in the left moon-shaped part. Finally, that means that $70 - 48 - 20 = 2$ people have a job lined up for after graduation, but never had an internship. That goes in the right moon-shaped part.

The Venn Diagram is in the next box.

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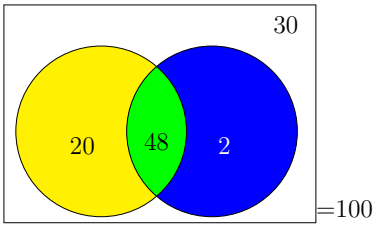
Here is the Venn Diagram for the previous example.

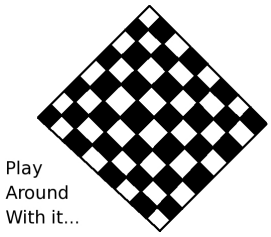




It is important to check your work in all problems, but in a Venn Diagram problem it is particularly easy. We have to verify that each of the data points that were given actually matches what we have in our diagram.

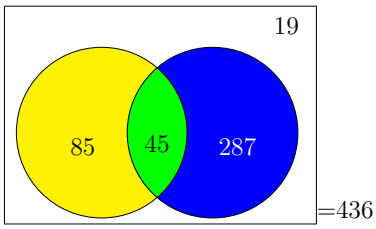
- We are told there are 100 people in the survey. We compute $20 + 2 + 48 + 30 = 100$. Good.
- We were told that 30 people have neither a job lined up nor have had an internship. That matches our diagram. Good.
- We were told that 68 people have had internships. We compute $20 + 48 = 68$. Good.
- We were told that 48 people have a job lined up and have had internships. That matches our diagram. Good.

Everything matches, so we know that we have solved the problem absolutely perfectly. By the way, we'll see this problem again, on Page 267.



1-2-6

The following Venn Diagram represents data collected from the IT Department of a major corporation about how they were contacted to respond to a problem. Some "trouble tickets" are handled via email, some on the phone, some require both email and the phone, and some are handled by other means (such as "in person" visits). The right circle represents phone contacts, and the left circle represents email contacts.



- How many problems were resolved by use of the phones?
- How many problems were resolved by use of email?
- How many problems were resolved by either phones or email?
- How many problems were resolved by neither phones nor email?
- How many problems were resolved by both phones and email?

The answers will be found on Page 77.

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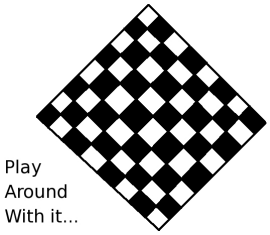
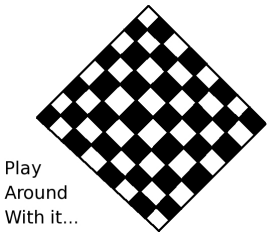


1-2-7

A survey of senior citizens is taken to see how much of the internet they use. Out of 106 seniors surveyed, 53 use e-mail regularly, and 40 use the web regularly. Furthermore, 34 use both regularly. Summarize this data in a Venn Diagram (put email on the left and the web on the right) and then answer: how many seniors in the survey regularly use neither email nor the web?

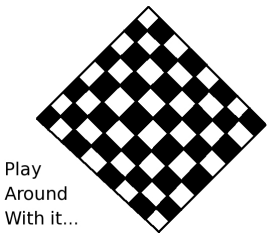
[Answer: to be found on Page 77.]



1-2-8

Suppose a large university gets a major donation and is thinking of opening either a college of nursing or a college of pharmacy with the new funds. A survey is taken of 486 undergraduates from biology, to see if they would stay on after graduation and enter one of those graduate colleges. Of those, 280 said that they would enter neither. Of the remainder, 106 said that they would consider nursing, and 26 of those said they would consider both. Make a Venn Diagram to summarize the situation. (Put nursing on the left and pharmacy on the right.) Now answer, how many biology students would consider a graduate degree in pharmacy?

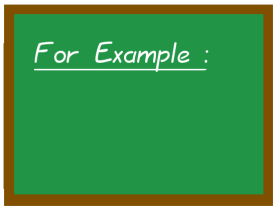
[Answer: to be found on Page 77.]



1-2-9

Imagine that you are hired as a consultant by a private prison that is surveying prisoners who have been convicted of their first murder. There are 85 convicts in the survey, and 79 of these had been convicted earlier of a drug offense. Of those, 58 had been convicted of a handgun-related crime. There were only 4 people convicted of a handgun-related crime but not a drug-related crime. Construct a complete Venn Diagram of the situation and then tell me how many of the convicts have neither a drug-related nor a handgun-related conviction? (Place drugs on the right and handguns on the left.)

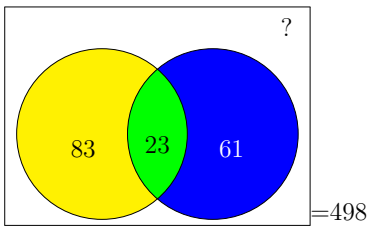
[Answer: to be found on Page 78.]



1-2-10

Imagine yourself tracking pet ownership in a homeowner's association. Out of 498 people, 106 have cats, and 84 have dogs. Furthermore, 23 people are raising both cats and dogs. How many people have a pet?

First, we note that 23 people raising both belong in the middle football-shaped part of the diagram—we write that first. Then let the left circle be cats, and the right circle be dogs. In the cat circle, we have $106 - 23 = 83$ people who have a cat and not a dog, so they go in the left moon-shaped part. Then there are $84 - 23 = 61$ people who have a dog but not a cat, and they belong in the right moon-shaped part. Now, looking at the diagram



you can see that $83 + 23 + 61 = 167$ people have a pet.

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Let's check our work in the previous example.

- We were told that 106 people have cats. We check $83 + 23 = 106$. Good.
- We were told that 84 people have dogs. We check $23 + 61 = 84$. Good.
- We were told that 23 people have both cats and dogs. That matches our diagram. Good.
- We had no use of the datum that 498 people were in the survey. That's a point which we will explore in the next box. For now, we ignore this one.

Since all the sums match what they should, we now can be absolutely certain that we did the problem correctly. Now we'll address the issue of the 498 in the next box.

In the previous example, note that we made no use of the 498, and furthermore, we have no need to calculate what goes in the background of the Venn Diagram. That's why I put a question-symbol (?!?) in the background region.

In order to be efficient on exams, it is important to calculate only what you need, and not anything extraneous. Otherwise you will lose valuable time. However, if you are curious

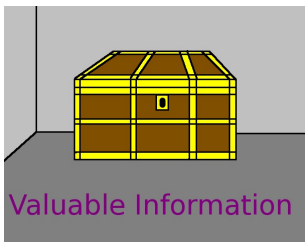
$$498 - 83 - 61 - 23 = 331$$

people have no pet. We could then put 331 in the background, where the question symbol ?!? is currently.



Sometimes we want to talk about the set of things *without* some property, rather than with it. For example, if a set $\mathcal{D}$ in a Venn Diagram, such as in the last problem, represents people who have dogs, then $\mathcal{D}^c$ means people not in $\mathcal{D}$. In other words, people in the survey that do not have dogs.

In a medical problem, if there is a set of patients, and $\mathcal{F}$ represents those that are female, then $\mathcal{F}^c$ would be those that are not female. We will have frequent use of this concept, called a *complement*. Sometimes it is easier to calculate the complement of a thing than to calculate the actual thing.



Recall from Page 58, that when you are working on a Venn Diagram problem, you have a set $\mathcal{U}$ that is the *universe* or *universal set* of the problem. That is a large set containing the entire problem. Then for any subset $\mathcal{S} \subseteq \mathcal{U}$, we write $\mathcal{S}^c$ to indicate those things that are in $\mathcal{U}$ but that are not found in $\mathcal{S}$. This is called the *complement* of $\mathcal{S}$.



Different textbooks will write the complement in different ways. For example, it can be F^c, $\sim F$, or F', and sometimes $\overline{F}$.

We'll use the F^c notation in this book, but I just thought that I should let you know about these other symbols—that way, if you see something in a different textbook, you will not be confused.

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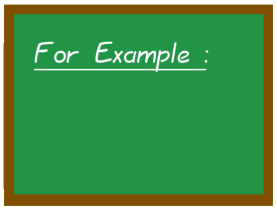
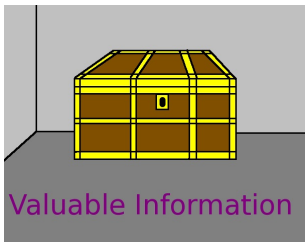
There is a distinction that some readers are familiar with, and some readers have never imagined. It has to do with a word that we use every day, a word that we've used for all of our lives. It's the word "or," and it has two uses: the *inclusive or* and the *exclusive or*.



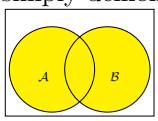
1-2-11

Let's look at four very simple cases of the word "or" as used in ordinary English. You'll see that your intuition and knowledge of English will make the distinction between the *inclusive or* and the *exclusive or* simple and perhaps even obvious.

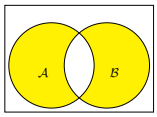
- When registering for courses at some university, there is a rule: to take ECON-425, you must have ECON-201 or ECON-210. Given these rules, if a student has taken both ECON-201 and ECON-210, can he take ECON-425? Of course! Therefore, the "or" *includes* the case of both.
- When buying food at the company cafeteria, there is a sign that says you can get a free salad or a free dessert with your main dish. Can you take both a free salad or a free dessert? Of course not! Therefore, the "or" *excludes* the case of both.
- The employee handbook at a fast-food restaurant says that if you're caught stealing from the register, or caught spitting in the food, then you're automatically fired. If I were working there, and if I were caught doing both of those things, would I be fired? Of course I would! Therefore, the "or" *includes* the case of both.
- Suppose Fred asks his wife, "Do you think I can wear shorts with this shirt?" She replies "No. You should wear jeans or slacks." Does her recommendation include Fred wearing both jeans and slacks simultaneously? Of course not! Her use of the word "or" *excludes* the case of both.



When someone says "$\mathcal{A}$ or $\mathcal{B}$," the distinction of the *inclusive or* versus the *exclusive or* can be simply demonstrated with a Venn Diagram:



The Inclusive Or
$A \cup B$



The Exclusive Or
$A \oplus B$

In mathematics, when we use the word "or," we always mean the *inclusive or* ($\cup$), unless otherwise noted.

The last sentence of the previous box is really important. Please read it again.

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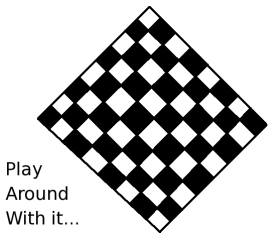
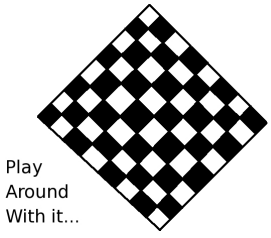


1-2-12

Classify the use of the word "or" in each of these sentences as inclusive versus exclusive.

1. At the cafeteria, there is a sign that says "due to allergies, none of our products have peanuts or shellfish."
2. A bottle of industrial solvents says that "this bottle may explode violently if exposed to high temperatures or high pressures."
3. There is a business trip that your boss wants you to participate in, but you don't want to go, because it is to an unpleasant location. She says to you "I don't care if it is unpleasant—go, or I'm firing you."
4. A particular university has a rule that if a professor is found to be teaching while drunk or while stoned, then the professor must be fired.

The answers will be on Page 78.

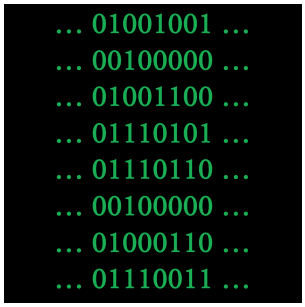


1-2-13

Classify the use of the word "or" in these sentences as inclusive versus exclusive.

5. Suppose there is a rule that to register for a specific class on music education, a student must be either a music major or an education major.
6. You are ordering a shipment via the telephone, and are going to pay with your company's credit card. The sales clerk asks "is it a visa or mastercard?"
7. A sign at the airport says that "it is illegal to carry guns or knives aboard the aircraft."
8. Your spouse looks over your family's financial situation. You are told "Unfortunately, we simply cannot afford to go to New York or London this year."

The answers will be on Page 78.



As you can see, English has some ambiguities. The meaning of the word "or" can be different depending on the context—inclusive in some cases, and exclusive in other cases.

As humans, we can use intuition to resolve the ambiguity. After a few seconds of thought, it is usually pretty clear whether the inclusive versus exclusive is meant. However, computers, databases, and other electronic hardware are not human. They do not have intuition. They must be programmed according to clear-cut rules.

For this reason, it is understandable that set theory, logic, and boolean algebra (a term which I have not yet defined) were chosen as the foundational concepts of computer science in the period when computers were first being built (the late 1940s to the early 1970s).



Earlier, on Page 46, I made the claim that set theory is a language. We're now going to practice that language by translations. We are going to convert set-theory notation to ordinary English, and ordinary English into set theory.

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For Example :

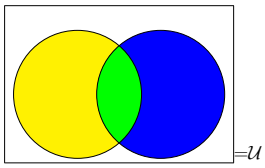
1-2-14

A survey of college professors was performed. Let $\mathcal{M}$ be those who are married, and let $\mathcal{T}$ be those who are tenured. What, in words, do the following sets indicate?

$$\mathcal{M} \cap \mathcal{T}, \mathcal{M}^c, \mathcal{M}^c \cap \mathcal{T}^c, \mathcal{M} \cup \mathcal{T}$$

- $\mathcal{M} \cap \mathcal{T}$ represents married tenured faculty.
- $\mathcal{M}^c$ represents unmarried faculty.
- $\mathcal{M}^c \cap \mathcal{T}^c$ represents unmarried untenured faculty.
- $\mathcal{M} \cup \mathcal{T}$ represents faculty that are either married or tenured.
- Remember, in mathematics, we do not say "either married, tenured or both," because the inclusive-or is assumed unless otherwise noted.

Continuing with the previous example, look at the following empty Venn Diagram. Let $\mathcal{M}$, the married faculty, be the left circle; let $\mathcal{T}$, the tenured faculty, be on the right.



- $\mathcal{M} \cap \mathcal{T}$ is the green football-shaped region.
- $\mathcal{M}$ is both the yellow moon-shaped region on the left and the green football-shaped region, taken together. It is the entire left circle.
- $\mathcal{M}^c$ is everything other than $\mathcal{M}$. It includes the blue moon-shaped region on the right, and the white background.
- $\mathcal{T}$ is both the blue moon-shaped region on the right and the green football-shaped region, taken together. It is the entire right circle.

We will continue in the next box.

Continuing from the previous box,

- $\mathcal{T}^c$ is everything other than $\mathcal{T}$. It includes the yellow moon-shaped region on the left, and the white background.
- $\mathcal{M}^c \cap \mathcal{T}^c$ represents everything in neither circle—the white stuff outside both circles.
- $\mathcal{M} \cup \mathcal{T}$ represents stuff in either of the circles—yellow, blue, and green taken together. It looks like the Mastercard logo.

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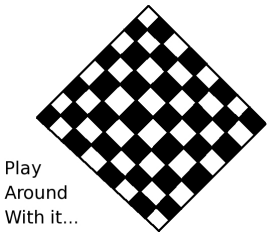
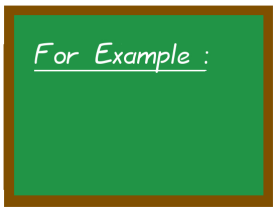


1-2-15

Let $\mathcal{U}$ represent the set of universities in the USA. Let $\mathcal{V}$ indicate those that are private, and let $\mathcal{N}$ represent those that offer the PhD degree. For this problem, we will consider all universities to be either public or private. Of course, no university is both public and private.

- What does $\mathcal{V}^c$ indicate? [Answer: Public universities.]
- What does $\mathcal{N}^c$ indicate? [Answer: Universities that cannot confer the PhD degree.]
- What does $\mathcal{V} \cap \mathcal{N}$ indicate? [Answer: Private PhD-granting universities.]
- What does $\mathcal{V}^c \cap \mathcal{N}$ indicate? [Answer: Public PhD-granting universities.]



1-2-16

Suppose one is investigating prisoners in a particular state—call the set of them $\mathcal{P}$. Let $\mathcal{M}$ indicate male prisoners, and let $\mathcal{F}$ indicate female prisoners. (Our model does not take into account transgender prisoners, unfortunately. Perhaps the model was constructed some number of years ago.) Let $\mathcal{V}$ indicate those in prison for at least one violent crime, and $\mathcal{N}$ be those in prison only for non-violent crimes. In other words, $\mathcal{V}$ and $\mathcal{N}$ are disjoint. (The word disjoint was first defined on Page 37.)

In the next box, we are now going to translate some set-theory notation into ordinary English, to practice our mastery of this new symbolic notation.

Referring back to the previous example,

- Before we start, since all prisoners are either violent or non-violent, and no prisoner is both, $\mathcal{N} = \mathcal{V}^c$ and $\mathcal{V} = \mathcal{N}^c$.
- The meaning of $\mathcal{M} \cap \mathcal{F}$ indicates "prisoners who are both male and female simultaneously." Since our model does not take into account any transgender prisoners, some of whom might self identify as *gender fluid*, we must realize that this is the empty set, $\{\}$.
- The meaning of $\mathcal{F} \cap \mathcal{V}$ indicates "violent female prisoners."
- The meaning of $\mathcal{M} \cap \mathcal{N}$ indicates "non-violent male prisoners."
- The meaning of $\mathcal{N} \cap \mathcal{P}$ indicates the meaning of $\mathcal{N}$, because $\mathcal{N}$ is a subset of $\mathcal{P}$, and thus intersecting $\mathcal{N}$ with $\mathcal{P}$ leaves $\mathcal{P}$ unchanged. More simply, the people in $\mathcal{N} \cap \mathcal{P}$ are the people in $\mathcal{N}$ because all people in $\mathcal{N}$ are by default also in $\mathcal{P}$. Thus we write "non-violent prisoners."
- The meaning of $\mathcal{N}^c$ indicates "prisoners who are not non-violent," and that is $\mathcal{V}$. This is an example of a double-negative. The prisoners who are not non-violent are the violent prisoners.

Note: In general $(\mathcal{S}^c)^c = \mathcal{S}$, for any set $\mathcal{S}$.

We will continue in the next box.

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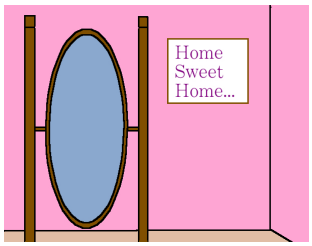
Continuing with the previous box, we translate some more complicated ideas now.

- The meaning of $\mathcal{F} \cup \mathcal{V}$ indicates "prisoners who are violent, female, or both."
- The meaning of $(\mathcal{M} \cap \mathcal{N})^c$ should be ascertained as a two step process. First, the meaning of $(\mathcal{M} \cap \mathcal{N})$ is "non-violent male prisoners" as we saw a few bullets ago. Thus, the complement of $(\mathcal{M} \cap \mathcal{N})$ is anyone who is in the problem's universe $\mathcal{P}$ but not in that set. This indicates the set of "prisoners who are either female or violent, or both," and we saw in the previous bullet that was $\mathcal{F} \cup \mathcal{V}$.
- The meaning of $\mathcal{M} \cup \mathcal{V}^c$ is easier obtained by first realizing that $\mathcal{V}^c$ is $\mathcal{N}$. That is to say, the complement of the violent prisoners is the set of non-violent prisoners. Thus $\mathcal{M} \cup \mathcal{V}^c$ is really $\mathcal{M} \cup \mathcal{N}$, the "prisoners who are either male or non-violent."
- Remember, we don't say "prisoners who are either male, non-violent, or both" in the previous bullet, because in mathematics, the inclusive-or is assumed, unless otherwise noted.

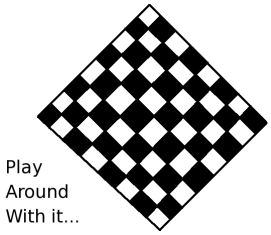


A Pause for Reflection...

Having explored two-circle Venn Diagrams to a great extent, we will now introduce three-circle Venn Diagrams. These are almost the same concept, but they allow us to organize more complex data.

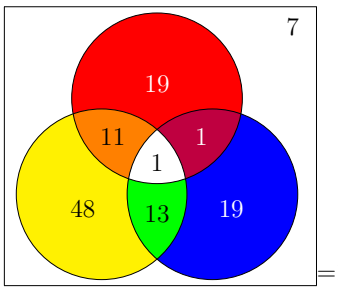
While two-circle Venn Diagrams are handy for displaying data about problems, many of the problems solvable with them could be solvable without them. Some of the problems that we did earlier could have been solved by intuition alone.

In contrast, I think you'll find that the three-circle Venn Diagrams are indispensable, especially when we get to the most advanced problems.



1-2-17

A survey was done of employees in a medium-sized office building in New Orleans to find out what type of music they'd like to be played in the office elevator. The survey had three choices: Classical Music, Jazz Music, and Rock Music. The data is summarized in the following Venn Diagram:



=119

The top circle indicates Rock music, the lower-left circle indicates Jazz music, and the lower-right circle indicates Classical music.

Based off of this diagram, please answer the questions in the following box.

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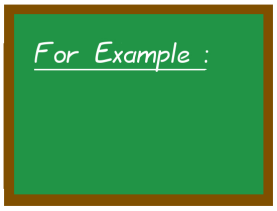
Referring back to the previous box,

- How many people indicated that they like Jazz?
- How many people indicated that they like Classical?
- How many people indicated that they like Rock?
- How many people indicated that they like only one type of music?
- How many people dislike rock music?
- How many people dislike classical music?
- How many people like both jazz and classical?
- How many people like both rock and classical?

The answers can be found on Page 79.



1-2-18

Let's imagine that you are interning with a company that makes clothing and backpacks for a 5th grade to 9th grade audience. You suggest they adopt an online advertising plan, and they commission a survey of middle school students, to discover which famous websites they use. Out of 1191 students who responded, 807 use Facebook, 707 use YouTube, and 908 use iTunes. Furthermore, 526 use both Facebook and YouTube, 667 use YouTube and iTunes, and 593 use Facebook and iTunes. Last but not least, 489 use all three. Fill in a Venn Diagram that represents all this data. (I strongly suggest that you start by drawing a 3-circle Venn Diagram on a piece of blank paper, and don't be worried if you have to read this 2–3 times before you get the swing of it.)

Okay, first off, only one data point represents something in the Venn Diagram without further calculation, and that is the "all three" category. The center bit gets 489. Next, we're going to go after those who use 2-out-of-3, as follows

- for the part between Facebook and YouTube but not iTunes, we write $526 - 489 = 37$.
- for the part between Facebook and iTunes but not YouTube, we write $593 - 489 = 104$.
- for the part between iTunes and YouTube but not Facebook, we write $667 - 489 = 178$.

These new numbers (37, 104, 178) go in the three mitre-shaped regions. We will continue our work in the next box.

We're continuing now with the problem from the previous box. Next, we have enough data now to figure out the single-website users:

- For Facebook only, we get $807 - 37 - 104 - 489 = 177$.
- For YouTube only, we get $707 - 37 - 178 - 489 = 3$.
- For iTunes only, we get $908 - 104 - 178 - 489 = 137$.

Now these numbers (177, 3, 137) go into the moon shaped regions (or if you like, regions shaped like a twice-bitten circular cookie). Finally, we can find out how many people don't use any of these sites at all. That comes out to

$$1191 - \underbrace{37 - 104 - 178}_{\text{users of 2}} - \underbrace{177 - 3 - 137}_{\text{users of 1}} - \underbrace{489}_{\text{users of 3}} = 66$$

The final Venn Diagram can be found in the next box.

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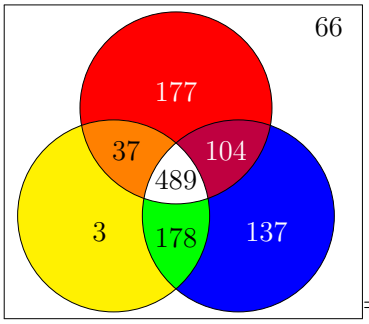
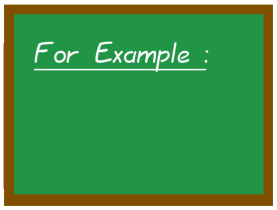
Here's the Venn Diagram for the problem of the previous two boxes. The red or top region is for Facebook; the yellow or left region is for YouTube; the blue or right region is for iTunes.



=1191

Okay, that was really hard, let's do another example together, before we go forward with a few checkerboard boxes. Here's a hint: aim for the center, and work outward.



1-2-19

A political action group in New York City is interested in reaching a younger generation of New Yorkers, who among other things, don't read newspapers. They commission a survey of some political-science students from various universities across the five boroughs. You've been hired as an intern to do the data analysis. The first thing you see is that three websites predominate in the data set. Namely, `nytimes.com`, `cnn.com`, and `nydailynews.com`. There are 162 people in the survey, and 63 read `nytimes.com`, while 42 read `cnn.com`, and 66 read `nydailynews.com`. The survey also discovers that 5 read all three. There were 13 people who read both `cnn.com` and `nytimes.com`, while 17 people read both `nytimes.com` and `nydailynews.com`. Also, 14 people read both `nydailynews.com` and `cnn.com`.

You are asked to make a Venn Diagram displaying all the data, and after that, to determine how many people read zero of, one of, two of, or three of these three specific news sites. We will solve the problem in the next box.

Before we start solving the example of the previous box, I suggest you draw a blank three-circle diagram on a piece of paper, and carefully follow along.

First, we see that 5 people read all three sites, so they go into the center. Let `nydailynews.com` be the top circle, while `cnn.com` and `nytimes.com` are the left and the right, respectively. Now it is easy to figure out the three regions that are shared by two but not three sites:

- Among `cnn.com` and `nytimes.com` but not `nydailynews.com`, we have $13 - 5 = 8$ people.
- Among `cnn.com` and `nydailynews.com` but not `nytimes.com`, we have $14 - 5 = 9$ people.
- Among `nydailynews.com` and `nytimes.com` but not `cnn.com`, we have $17 - 5 = 12$ people.

We will continue in the next box.

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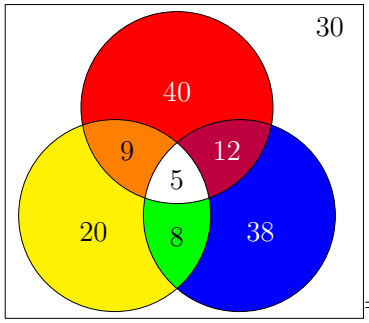
We're continuing now the problem from the previous box. The data about people who read exactly two news sites, in turn, allows us to find out who reads exactly one news site:

- Those who read `nytimes.com` alone are $63 - 8 - 5 - 12 = 38$.
- Those who read `nydailynews.com` alone are $66 - 12 - 5 - 9 = 40$.
- Those who read `cnn.com` alone are $42 - 8 - 5 - 9 = 20$.

And finally, we can figure out who does not read any of these three. That would include people who read other websites (such as `wsj.com`) or those who do not read the news at all. Here is the calculation:

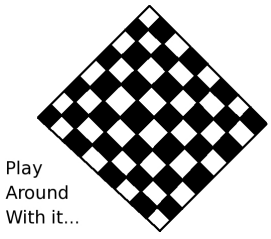
$$162 - \underbrace{38 - 20 - 40}_{\text{1 paper}} - \underbrace{12 - 8 - 9}_{\text{2 papers}} - \underbrace{5}_{\text{3 papers}} = 30$$

This is the Venn Diagram for the problem in the previous box. The red or top region is for `nydailynews.com`; the yellow or left region is for `cnn.com`; the blue or right region is for `nytimes.com`. Of course, it is possible that your diagram has them in different places.



=162

Believe it or not, we are not yet finished. The problem continues in the next box.



1-2-20

Resuming with where we left off in the previous box, we were asked to find out how many people read zero of, one of, two of, and three of these three particular news sites. You can easily do this yourself, looking at the Venn Diagram in the previous box.

- How many people read 0 of these 3 news sites? [Answers: 30 people.]
- How many people read 1 of these 3 news sites? [Answers: 98 people.]
- How many people read 2 of these 3 news sites? [Answers: 29 people.]
- How many people read all 3 of these news sites? [Answers: 5 people.]

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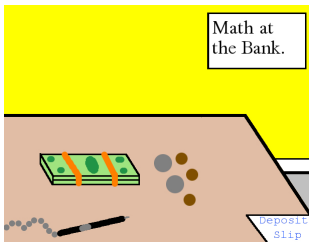
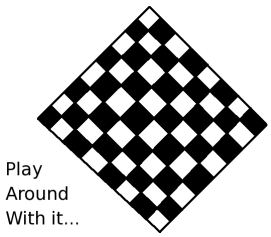


The three moon-shaped regions of the Venn Diagram (the ones that look like a circular cookie with two bites taken out of it) have a special interpretation in business. Consider the previous example.

- Suppose they were advertising on `nytimes.com`, and `cnn.com`, but not `nydailynews.com`. That "40" indicates that they would gain 40 new customers by adding that website. This might be rather attractive, if the ads are priced cheap enough.
- Conversely, if they were advertising on `nydailynews.com`, and `nytimes.com`, but not `cnn.com`. That "20" indicates that they would gain only 20 new customers by adding that website. That is considerably less attractive.

This idea will reappear in a problem on Page 109.

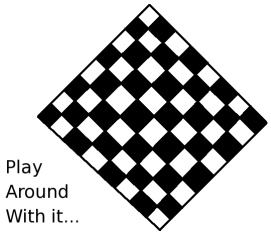


1-2-21

Now I'd like to ask you to repeat that (rather long) problem about the three news websites. However, use the following new values instead of the old values.

There are 644 people in the survey, and 155 read `nytimes.com`, while 322 read `cnn.com`, and 213 read `nydailynews.com`. The survey also discovers that 32 read all three. There were 56 people who read both `cnn.com` and `nytimes.com`, while 48 people read both `nytimes.com` and `nydailynews.com`. Also, 80 people read both `nydailynews.com` and `cnn.com`.

Once again, you are asked to make a Venn Diagram displaying all the data, and determine how many people read zero, one, two, and three news sites. The solution can be found on Page 79.

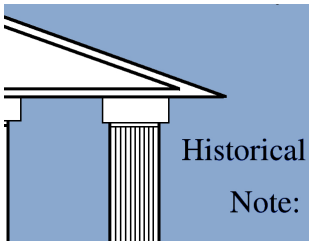


1-2-22

Just for practice I'd like to ask you to repeat the problem about surveying middle school students (from Page 69), with the following values instead.

Out of 543 who responded, 444 use Facebook, 308 use YouTube, and 304 use iTunes. Furthermore, 234 use both Facebook and YouTube, 239 use YouTube and iTunes, and 237 use Facebook and iTunes. Last but not least, 189 use all three.

Fill in a Venn Diagram that represents all this data. The solution can be found on Page 79.



You might be wondering why these are called "Venn Diagrams." They are named for John Venn (1834–1923), an English philosopher and logician. His life story is actually rather interesting. He started his university studies at Gonville and Caius College, Cambridge, studying mathematics and philosophy.

The combination of mathematics and philosophy is startling to us today, but it was once more common, including Rene Descartes (1596–1650), Blaise Pascal (1623–1662), Gottfried Wilhelm Leibniz (1646–1716), Henri-Louis Bergson (1859–1941), Bertrand Russell (1872–1970), Kurt Gödel (1906–1978), and several others. Analytical philosophy (as opposed to continental philosophy) is based on logic, including the extremely rigorous "symbolic logic" that eventually would form the foundations, centuries later, of computer science and computer engineering. It was from his logic background that made John Venn realize that the diagrams would be useful in teaching not only logic, but in teaching probability.

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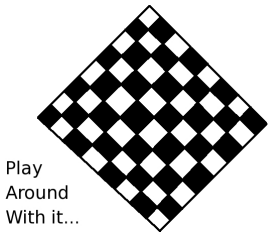
Continuing with the previous box, John Venn was ordained an Anglican priest in 1859, being descended from a very long ancestry of Anglican priests, stretching back to Vicar William Venn (1568–1621). However, he left his parish after only three years to become a lecturer at Cambridge. Venn also published an important textbook, *Symbolic Logic*, in 1881. He would later become president of Gonville and Caius College, Cambridge, where he first started his studies. To this day, there stands in their chapel a stained glass memorial to him, in the shape of a three-circle Venn Diagram.

Curiously, Venn diagrams were not invented by John Venn. They were instead invented by Euler, who discovered the number e and many other things. (See Euler's biographical sketch on Page 710, in the Historical Appendix "Fermat's Last Theorem and Some Famous Unsolved Problems.") In fact, Venn himself called them "Euler Diagrams." However, they are named after John Venn because he popularized their role in teaching. Also, many items in mathematics are already named for Euler.

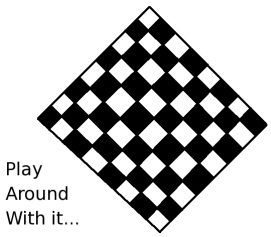


1-2-23

The survey below represents data drawn from college students about their weight-control habits. Draw a Venn Diagram to represent the data from the survey.

- 29 students play sports.
- 32 students work out at the gym.
- 7 students both play sports and diet. Of those, 2 do not work out at the gym.
- 10 students are both working out at the gym and on a diet.
- 27 students are dieting but do not work out at the gym nor play a sport.
- 10 students both work out at the gym and play a sport but do not diet.
- 24 students are neither working out at the gym, playing a sport, nor on a diet.
- 97 students took the survey.

The solution can be found on Page 80.



1-2-24

An insurance company is surveying travelers, to learn how much of their summer travel is taken with an airplane, a train or an automobile. The following data is known; make a complete Venn Diagram with all the data.

The number of people who flew was 1307. The number of people who both flew and used a train was 602. The people who used all three were 398 in number. Those who flew but didn't drive came to a total of 599. Those who drove but did not use a train totaled 1097. There were 610 people who used both trains and cars. The number of people who used either a car or a train or both was 2050. Lastly, 421 people used none of these (I guess that they either took a cruise or stayed home.)

You're asked how many people drove but used neither a train nor an airplane, and also, how many people were in the entire survey.

The Venn Diagram and answers are given on Page 80. Let the top circle represent air travel, the lower-left circle represent car travel, and the lower-right circle represent train travel.

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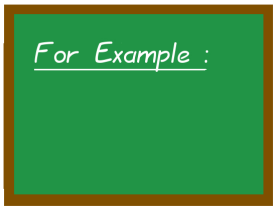
Sometimes, you might be presented with a Venn Diagram problem that is unsolvable. It can be unsolvable because there are many solutions, and you don't have enough data to reach the end of the problem; it can also be impossible, in the sense that no solution exists.

In real life, a Venn Diagram with no solution can represent a data collection error. In the classroom, this can happen because the instructor has made a mistake, or because the instructor wants to challenge your critical thinking skills. The case of multiple solutions is far more common, and happens relatively often when a survey or experiment is designed by someone other than an expert statistician.

I don't like impossible Venn Diagrams, and I never give them (intentionally) on tests. However, it is probably a good idea that you know that this type of thing exists, in case you see it during some research project or in industry. We will explore this point in greater detail on Page 130.



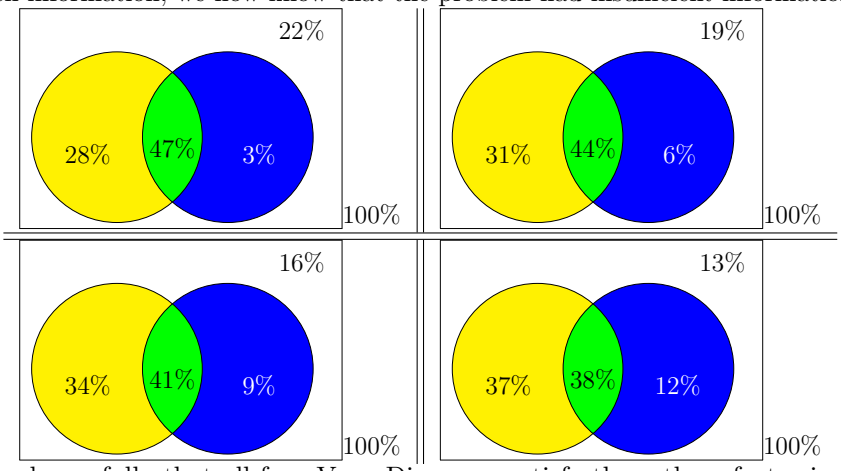
1-2-25

Here is an example of Venn Diagram problem that is not solvable, because of having not enough information provided. The following information is for a hypothetical software company that makes a website, which had an unfortunate bug for a month last summer. We're going to make a Venn Diagram of the following facts.

- 75% of the company's users experienced the bug.
- 50% of the company's users quit using the website.
- Don't forget that the Venn Diagram should add up to 100%.

Let the left circle indicate those users who encountered the bug, and the let the right circle indicate those users who quit using the website. In the next box, you will find four Venn Diagrams that one could write. Each of them is consistent with the previous three facts.

Here are four diagrams consistent with the facts of the previous box. Since they are different from each other and consistent with the given information, we now know that the problem had insufficient information and cannot be solved.



You should now check carefully that all four Venn Diagrams satisfy those three facts given above. With that in mind, we can tell that the original problem was not solvable because of insufficient information. Even with two Venn Diagrams that are different, but which both satisfy all the given facts, we would conclude that the problem cannot be solved.

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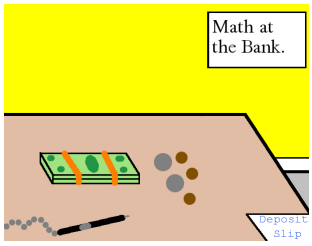
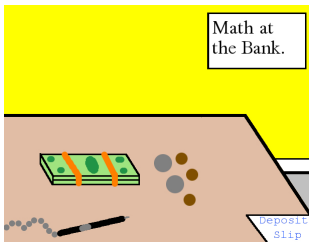


It is worthwhile to look at the Venn diagrams in the previous box, and to try to interpret them from a business perspective (in this case, a customer relations and retention perspective).

- Those in the left moon-shaped region represent customers who encountered the bug, but didn't quit using the website. We could say they forgave the website.
- Those in the football-shaped region represent customers who encountered the bug, and then quit using the website.
- Those in the right moon-shaped region represent customers who did not encounter the bug, but quit using the website anyway.
- Those in the background of the Venn Diagram represent users who neither encountered the bug nor who quit using the website—in other words, users that are happy (more or less) with the website.

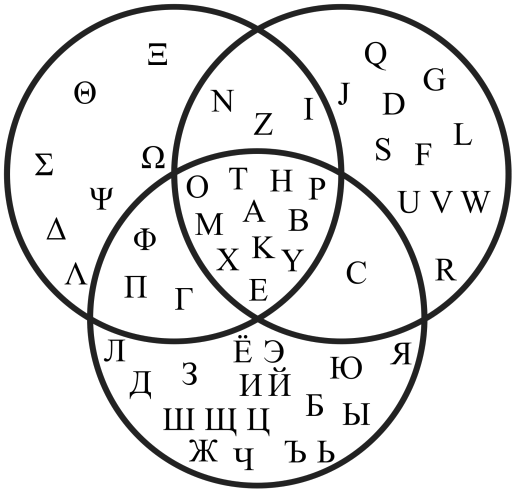
It is pretty clear that this software company has serious problems. We will continue in the next box.



Continuing with the previous box, it is surprising how little information we actually have. Based only on the four Venn Diagrams above, here are some measures of the uncertainty.

- The bug could be responsible for as many as $47/(47+3) = 94\%$ of lost users or as few as $38/(38+12) = 76\%$ of lost users.
- The website could have been forgiven by as many as $37/75 = 49.3\overline{3}\%$ of the users who found the bug or as few as $28/75 = 37.3\overline{3}\%$ of the users who found the bug.
- For both these questions, since the uncertainty is visible even in the most significant digit, we should say that we have zero digits (!) of precision.

As you might imagine, it would be very difficult to make any business decisions on the basis of this highly incomplete data.



There is another use of the Venn Diagrams that I have not yet mentioned. Venn Diagrams are often used in the humanities and social sciences, particularly to express complex relationships. Consider the diagram on the left. It hopes to explain how the Cyrillic Alphabet (the alphabet used to write the Russian language, and several others), the Greek Alphabet, and the English Alphabet have many letters in common. However, the alphabets are not identical nor are they close to identical.

In particular, in this diagram, English is represented by the upper-right circle; Russian is represented by the bottom circle; Greek is represented by the upper-left circle. For example, П is found in both Russian and Greek, but not English. Meanwhile, B is found in all three, and Z is found in Greek and English but not Russian, and so forth.

The image in the box above was created by Tilman Piesk, and uploaded to the WikiMedia Commons in 2011. He has released the image to the public domain, but I am happy to offer this academic citation and image credit anyway.

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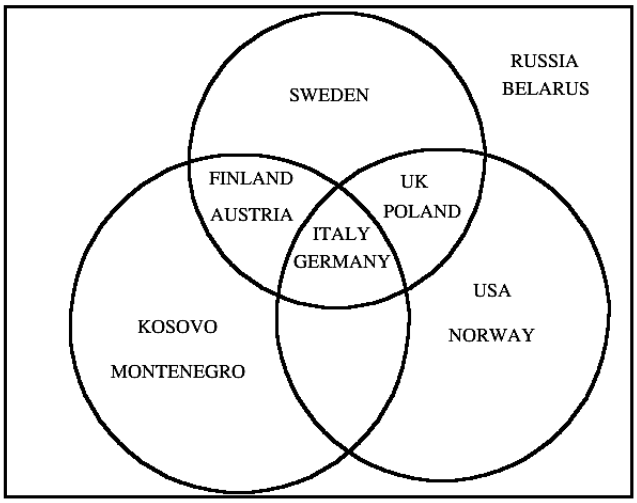


Here is another example, where I have taken a subset of the countries that are geographically part of the continent of Europe, plus the USA as well. I have only listed a few example countries, to keep the Venn Diagram from becoming crowded.

The top circle represents the member nations of the European Union. The lower-left circle represents nations that use the Euro. Already, here we have a point that confuses many students. I have encountered many students who imagine that the Euro is used in all European Union member states, and that only European Union member states use the Euro. As you can see, this is not true.

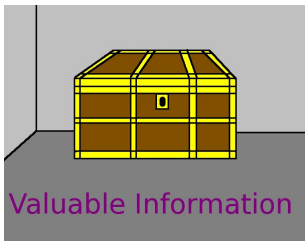
The lower-right circle represents NATO. As you can see, both Russia and Belarus have the distinction of not being in the EU, not being in NATO, and not using the Euro. For that reason, they are listed outside the three circles.

For completeness, I should note that the information in the previous box was accurate in January of 2015. In later years, alliances might shift and membership in some of those circles might change, or perhaps not.

For example, on June 23rd of 2016, the United Kingdom voted to leave the European Union. However, this exit of Britain from the EU (called "Brexit" in journalist slang), will take several years to be implemented. Therefore, I have not revised the diagram at this time.

You have now completed this module. All that remains is a listing of the answers to a few checkerboards from earlier in the module.

However, I have one important comment (in the next box) that is very important for helping you identify whether your answers are correct or incorrect.



By now, you've surely figured on your own that for any sets $\mathcal{A}$ and $\mathcal{B}$ that the following is always true

$$\mathcal{A} \cup \mathcal{B} = \mathcal{B} \cup \mathcal{A} \qquad \text{and also} \qquad \mathcal{A} \cap \mathcal{B} = \mathcal{B} \cap \mathcal{A}$$

Do keep that in mind when you're checking your work. Your answer might look slightly different from mine, because of the above reason.

This property is also true for addition $2 + 3 = 3 + 2$ and multiplication $(5)(4) = (4)(5)$. This property is called *commutativity*. For example, subtraction is not commutative, because

$$3 - 5 = -2 \qquad \text{and} \qquad 5 - 3 = 2 \qquad \text{but} \qquad -2 \neq 2$$

and likewise division is not commutative because

$$3 \div 5 = \frac{3}{5} \qquad \text{and} \qquad 5 \div 3 = \frac{5}{3} \qquad \text{but} \qquad \frac{3}{5} \neq \frac{5}{3}$$

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Here are the answers to the question (from Page 59) about HIPAA and FERPA training at a company that makes databases.

- How many employees have training in FERPA? [Answer: 30.]
- How many employees have training in HIPAA? [Answer: 32.]
- How many employees have training in both? [Answer: 13.]
- How many employees have training in either? [Answer: 49.]
- How many employees have training in neither? [Answer: 15.]

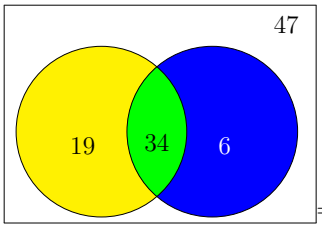


Here are the solutions to the question about email and phone use in the IT Department of a corporation, as given on Page 61.

- How many problems were resolved by use of the phones? [Answer: 332.]
- How many problems were resolved by use of email? [Answer: 130.]
- How many problems were resolved by either phones or email? [Answer: 417.]
- How many problems were resolved by neither phones nor email? [Answer: 19.]
- How many problems were resolved by both phones and email? [Answer: 45.]



Here is the solution to the question about senior citizens and the internet.

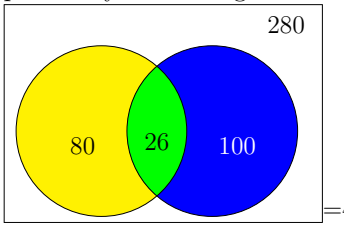


=106

We must not forget the ancillary question: 47 senior citizens neither use e-mail nor the web.



Below is the solution to the question about a large university opening either a college of pharmacy or a college of nursing.



=486

For the ancillary question: 126 undergraduates would consider graduate work in pharmacy.

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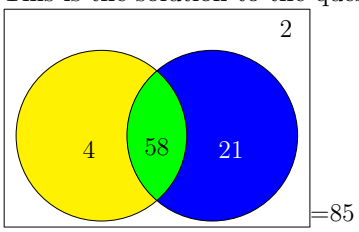
This is the solution to the question about convicts and their prior convictions.



=85

Next, for the ancillary question, 2 murderers had neither a drug-conviction nor a handgun-related conviction on their record.



Here are the answers to the first three sentences (from Page 65) involving the exclusive/inclusive or.

1. At the cafeteria, there is a sign that says "due to allergies, none of our products have peanuts or shellfish." Because the cafeteria could never serve a product that contains both peanuts and shellfish, products that contain both are prohibited. This is an inclusive or.
2. A bottle of industrial solvents says that "this bottle may explode violently if exposed to high temperatures or high pressures." If we expose that bottle to both high pressure and high temperature, will the bottle explode? Yes! Therefore, this is an inclusive or.
3. When your boss says "I don't care if it is unpleasant—go, or I'm firing you." Clearly, the situation of your going on the trip AND getting fired is not part of the arrangement. Therefore, this is an exclusive or.



Here are the answers to the last five sentences (from Page 65) involving the exclusive/inclusive or.

4. When the rule says "A particular university has a rule that if a professor is found to be teaching while drunk or while stoned, then the professor must be fired," clearly a professor who is simultaneously both drunk and stoned (while teaching) would be fired. Therefore, this is an inclusive or.
5. Suppose there is a rule that to register for a specific class on music education, a student must be either a music major or an education major. If a student is double majoring, taking both music and education, is he prohibited? Of course not! Therefore, this is an inclusive or.
6. The sales clerk asks "is it a visa or mastercard?" Since a credit card cannot be both a visa and a mastercard simultaneously, clearly this is an exclusive or.
7. A sign at the airport says that "it is illegal to carry guns or knives aboard the aircraft." Can I carry both? Absolutely not. Therefore, this is an inclusive or.
8. Your spouse looks over your family's financial situation. You are told "Unfortunately, we simply cannot afford to go to New York or London this year." You definitely cannot afford both, so this is an inclusive or.

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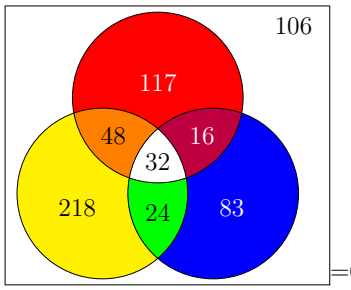
Here are the answers to the question about what music to play in elevators in an office building in New Orleans, given on Page 68.

- How many people indicated that they like Jazz? [Answer: 73.]
- How many people indicated that they Classical? [Answer: 34.]
- How many people indicated that they Rock? [Answer: 32.]
- How many people indicated that they like only one type of music? [Answer: 86.]
- How many people dislike rock music? [Answer: 87.]
- How many people dislike classical music? [Answer: 85.]
- How many people like both jazz and classical? [Answer: 14.]
- How many people like both rock and classical? [Answer: 2.]



Below, we have the solution to the problem about the three news sites, given on Page 72.



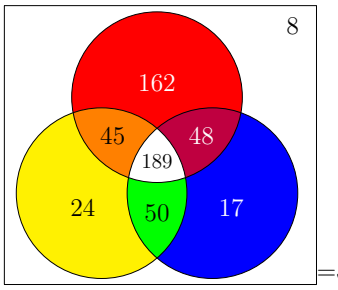
=644

As before, the red or top region is for `nydailynews.com`; the yellow or left region is for `cnn.com`; the blue or right region is for `nytimes.com`.

This problem had an ancillary question as well, about how many people use 0, 1, 2, or 3 sites. The answer is that 106 people read zero of these sites; 418 people read one site; 88 people read two sites; 32 people read all three sites.



Here is the solution to the problem about middle schoolers and the websites that they visit, as given on Page 72.



=543

As previously, the red or top region is for Facebook; the yellow or left region is for YouTube; the blue or right region is for iTunes.

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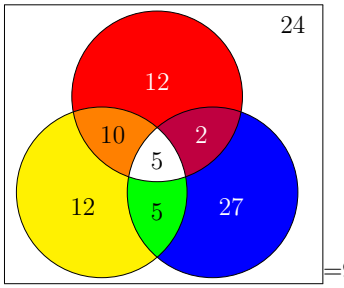
Here is the solution to the problem about college student's weight control habits, as given on Page 73.

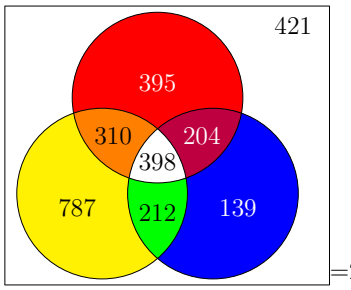


=97

In my solution, the top circle represents sports, the lower-left circle represents working out at the gym, and the lower-right circle represents dieting. However, your answer can have the circles in different positions. That's completely fine.



This is the Venn Diagram for the airplanes, trains and automobiles question on Page 73.



=2866

Remember, the top circle represents air travel, the lower-left circle represents car travel, and the lower-right circle represents train travel. We also have the important ancillary questions:

- How many people drove but did not use an airplane or a train? [Answer: 787.]
- How many people were in the survey? [Answer: 2866.]

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