Tutorial on interactive theorem proving in Lean

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Computers and mathematics

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An interactive theorem prover is a program that can reason with mathematical definitions, theorems and proofs, provided a user writes theses in a language that the program can understand.

This is called formalization.

Talk Topics

In this tutorial:

1. A brief introduction to Lean
2. Demo
3. Try out Lean yourself

Lean

Lean is an interactive theorem prover mainly
developed by Leonardo de Moura at Microsoft
Research.

It is open source, and under active development for 10 years.

mathlib

mathlib is the mathematical library of Lean.

It is a general-purpose: it contains algebra, topology, analysis, differential geometry, probability theory, category theory, combinatorics, logic, . . .

It is decentralized: contributors come with their own plans and goals.

It is large: mathlib has over 1 million lines of code, written by over 270 contributors.

It is active: There are more than 100 contributions every week and every contribution is reviewed by one of the 23 mathlib maintainers.

I have worked with Lean since 2014 and am a mathlib maintainer since 2019.

Selected Lean formalizations

- Spectral sequences (2017; using HoTT; van Doorn et al.)
- Definition of perfectoid spaces (2019; Buzzard, Commelin, Massot)
- Independence of the continuum hypothesis (2019; van Doorn, Han)
- Witt vectors (2020; Commelin, Lewis)
- Finiteness of the class group of a global field (2021; Baanen, Dahmen, Narayanan, Nuccio)
- Liquid tensor experiment (2022; Commelin et al.)
- Sphere eversion project (2022; van Doorn, Massot, Nash)

Why formalize mathematics?

1. Collaboratively create a unified mathematical library

2. Check proofs

3. Teach mathematics

4. Create mathematics

tinyurl.com/LeanTutorial