Report from Dagstuhl Seminar 19181

Computational Geometry

Edited by

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Abstract

This report documents the program and the outcomes of Dagstuhl Seminar 19181 "Computational Geometry". The seminar was held from April 28 to May 3, 2019 and 40 participants from various countries attended it. New advances and directions in computational geometry were presented and discussed. The report collects the abstracts of talks and open problems presented in the seminar.

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1 Executive Summary

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Computational Geometry

Computational geometry is concerned with the design, analysis, and implementation of algorithms for geometric and topological problems, which arise naturally in a wide range of areas, including computer graphics, CAD, robotics, computer vision, image processing, spatial databases, GIS, molecular biology, sensor networks, machine learning, data mining, scientific computing, theoretical computer science, and pure mathematics. Computational geometry is a vibrant and mature field of research, with several dedicated international conferences and journals and strong intellectual connections with other computing and mathematics disciplines.

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Seminar Topics

The emphasis of this seminar was on presenting recent developments in computational geometry, as well as identifying new challenges, opportunities, and connections to other fields of computing. In addition to the usual broad coverage of new results in the field, the seminar included broad survey talks on algebraic methods in computational geometry as well as geometric data structures. The former focus area has seen exciting recent progress and the latter is a fundamental topic at the heart of computational geometry. There are numerous opportunities for further cross-disciplinary impact.

Algebraic Methods in Computational Geometry

The polynomial method of Guth and Katz of 2010 has had a fundamental impact on discrete geometry and other areas, which was already envisioned by the talk of Jiří Matoušek at the Annual European Workshop on Computational Geometry in 2011, four years before he passed away. Indeed, the polynomial method has attracted the attention of many researchers, including famous ones like Janos Pach, Micha Sharir, and Terence Tao. Applications have been found not only in making progress on long-standing combinatorial geometry problems, but also in the design and analysis of efficient algorithms for fundamental geometric problems such as range searching, approximate nearest search, diameter, etc. The polynomial method is very powerful and it offers a new research direction in which many interesting new results can potentially be discovered.

Geometric Data Structures

Many beautiful results in geometric data structures have been established in the early days of the field. Despite of this, some long-standing problems remain unresolved and some of the recent progress is in fact made using the polynomial method mentioned previously. Independently, there have been some recent advances in our understanding of lower bounds and the usage of more sophisticated combinatorial constructions and techniques such as shallow cuttings, optimal partition trees, discrete Voronoi diagrams, etc. There are also new applications that require the modeling of uncertain data and hence call for a study of the performance of geometric data structures under a stochastic setting.

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Open problems

3 Overview of Talks

3.1 A Review of (some) Data Structure Lower Bound Techniques

Peyman Afshani (Aarhus University, DK)

In this talk, we will have a broad look at the landscape of data structure lower bounds. We will begin by introducing some fundamental lower bound models and then move on to demonstrate the key techniques that enable us prove non-trivial results in each model. These include the pointer machine model, the cell-probe model, the I/O-model, and the semi-group (or group) model. We will also very briefly touch the conditional lower bounds.

3.2 Hard problems in knot theory

Arnaud de Mesmay (University of Grenoble, FR)

Joint work of Arnaud de Mesmay, Yo'av Rieck, Eric Sedgwick, Martin Tancer
Main reference Arnaud de Mesmay, Yo'av Rieck, Eric Sedgwick, Martin Tancer: "The Unbearable Hardness of Unknotting", in Proc. of the 35th International Symposium on Computational Geometry, SoCG 2019, June 18-21, 2019, Portland, Oregon, USA., LIPIcs, Vol. 129, pp. 49:1–49:19, Schloss Dagstuhl – Leibniz-Zentrum fuer Informatik, 2019.
URL http://dx.doi.org/10.4230/LIPIcs.SoCG.2019.49

Quite a few problems in knot theory are extremely hard to solve algorithmically (like testing whether two knots are equivalent), and some of them are not even known to be decidable (like computing the unknotting number of a knot). However, very few hardness results are known. We show how a rather simple construction with Borromean rings can be leveraged to establish a handful of NP-hardness proofs for seemingly unrelated problems. Our main result shows that deciding if a diagram of the unknot can be untangled using at most k Reidemeister moves (where k is part of the input) is NP-hard. We also prove that several natural questions regarding links in the 3-sphere are NP-hard, including detecting whether a link contains a trivial sublink with n components, computing the unlinking number of a link, and computing a variety of link invariants related to four-dimensional topology (such as the 4-ball Euler characteristic, theslicing number, and the 4-dimensional clasp number).

3.3 Plantinga-Vegter algorithm takes average polynomial time

Alperen Ergür (TU Berlin, DE)

Joint work of Felipe Cucker, Alperen A. Ergür, Josué Tonelli-Cueto
Main reference Felipe Cucker, Alperen A. Ergür, Josué Tonelli-Cueto: "Plantinga-Vegter algorithm takes average polynomial time", CoRR, Vol. abs/1901.09234, 2019.
URL https://arxiv.org/abs/1901.09234

We provide smoothed analysis of an adaptive subdivision algorithm due to Plantinga and Vegter. The only available complexity analysis of this algorithm was due to Burr, Gao, Tsigaridas which provided worst case bounds that are exponential in the degree of the input equation. More in the line the practical success of PV algorithm, we provide polynomial bounds in terms of the degree.

3.4 General Polynomial Partitionings and their Applications in Computational Geometry

Esther Ezra (Georgia Tech – Atlanta, US & Bar-Ilan Univ. Ramat Gan, IL)

Joint work of Pankaj Agarwal, Boris Aronov, Esther Ezra, Joshua Zahl
Main reference Pankaj Agarwal, Boris Aronov, Esther Ezra, Joshua Zahl: "An Efficient Algorithm for Generalized Polynomial Partitioning and Its Applications", CoRR, Vol. abs/1812.10269, 2018.
URL https://arxiv.org/abs/1812.10269

Since the celebrated work of Guth in Katz on the Erdos distinct distances problem, polynomial partitioning became a central tool in solving incidence problems, as well as other main problems in discrete geometry. In spite of this progress, the application of polynomial partitioning in solving computational problems received considerably less attention.

Polynomial partitioning for a set of geometric objects forms a space decomposition, such that any component in this decomposition is intersected by a small fraction of the input objects. In this talk, I will survey the polynomial partitioning technique by first presenting the setting of points in d-space, addressed by Guth and Katz, and then discussing polynomial partitioning for general semi-algebraic sets, studied by Guth. I will then describe the algorithmic issues concerning the construction of such polynomials. Whereas there are efficient algorithms to construct polynomial partitionings of the first kind, it is currently unknown how to effectively construct general polynomial partitionings. I will present an efficient algorithm that constructs a general polynomial partitioning for semi-algebraic sets in d-space, which, as a main tool, exploits the concept of "quantifier elimination" combined with "epsilon-approximations". The running time of this algorithm is only linear in the number of input objects. As a preliminary result, I will present an algorithm that constructs a space decomposition for a collection of algebraic curves in 3-space, with complexity bounds similar to those of Guth. These results have several algorithmic implications, including a nearly-optimal algorithm to eliminate depth cycles among disjoint triangles in 3-space, an efficient range-search mechanism in the fast-query/large-storage regime, and an efficient point-location machinery that outperforms traditional point-location machineries exploiting vertical decompositions.

3.5 Approximating the Geometric Edit Distance

Kyle Jordan Fox (University of Texas – Dallas, US)

Joint work of Kyle Jordan Fox, Xinyi Li

We describe the first sublinear approximate strictly subquadratic time algorithms for computing the geometric edit distance of two point sequences in constant dimensional Euclidean space. First, we present a randomized $O(n \log^2 n)$ time $O(\sqrt{(}n))$-approximation algorithm. Then, we generalize our result to give a randomized alpha-approximation algorithm for any alpha in $[1, \sqrt{(}n)]$, running in time $\tilde{O}(n^2/\alpha^2)$. Both algorithms are Monte Carlo and return approximately optimal solutions with high probability.

3.6 Geometry and Generation of a New Graph Planarity Game

Wouter Meulemans (TU Eindhoven, NL)

Joint work of Rutger Kraaijer, Marc van Kreveld, Wouter Meulemans, André van Renssen
Main reference Rutger Kraaijer, Marc van Kreveld, Wouter Meulemans, André van Renssen: "Geometry and Generation of a New Graph Planarity Game", in Proc. of the 2018 IEEE Conference on Computational Intelligence and Games, CIG 2018, Maastricht, The Netherlands, August 14-17, 2018, pp. 1–8, IEEE, 2018.
URL https://doi.org/10.1109/CIG.2018.8490404

We introduce a new abstract graph game, Swap Planarity, where the goal is to reach a state without edge intersections and a move consists of swapping the locations of two vertices connected by an edge. We analyze this puzzle game using concepts from graph theory and graph drawing, computational geometry, and complexity. Furthermore, we specify what good levels look like and we show how they can be generated. We also report on experiments that show how well the generation works.

3.7 Multipoint evaluation for the visualization of high degree algebraic surfaces

Guillaume Moroz (INRIA Nancy – Grand Est, FR)

Joint work of Guillaume Moroz, Marc Pouget

The surface solution of a polynomial equation $f(x, y, z) = 0$ can be visualized using for example the marching cube algorithm. This requires to evaluate f on a grid of points in R^3. In this talk, we will review the existing methods to compute the evaluation of a polynomial on multiple points and we will show how some of these methods can be adapted to visualize efficiently algebraic curves and surfaces of degree ranging from 10 to 400.

3.8 Innovations in Convex Approximation and Applications

David M. Mount (University of Maryland – College Park, US)

Joint work of Ahmed Abdelkader, Sunil Arya, Guilherme da Fonseca, David M. Mount

Recently, new approaches to convex approximation have produced major improvements to approximation algorithms for a number of geometric optimization and retrieval problems. These include computing the diameter and width of a point set, kernels for directional width, bichromatic closest pairs, Euclidean minimum spanning trees, and nearest neighbor searching under various distance functions including the Mahalanobis distance and Bregman divergence. In this talk, I will describe these techniques, including Macbeath regions, Delone sets in the Hilbert metric, and convexification, and I will explain how these techniques can be applied to obtain these improvements.

3.9 Geodesic Voronoi Diagrams in Simple Polygons

Eunjin Oh (MPI für Informatik – Saarbrücken, DE)

Main reference Eunjin Oh: "Optimal Algorithm for Geodesic Nearest-point Voronoi Diagrams in Simple Polygons", in Proc. of the Thirtieth Annual ACM-SIAM Symposium on Discrete Algorithms, SODA 2019, San Diego, California, USA, January 6-9, 2019, pp. 391–409, SIAM, 2019.
URL https://doi.org/10.1137/1.9781611975482.25

In the presence of polygonal obstacles, the distance of two points is measured by the length of a shortest path between the two points avoiding obstacles. In this talk, I introduce several recent results on problems defined in polygonal domains including an $O(n + m \log m)$-time algorithm for computing the geodesic Voronoi diagram of m points in a simple n-gon.

3.10 Intersection patterns of sets in the plane

Zuzana Patáková (IST Austria & Charles University Praha)

Joint work of Gil Kalai, Zuzana Patáková

Helly theorem states that to decide whether a finite family of convex sets in $\mathbb{R}^d$ has a point in common, it is enough to test only intersections of d+1 sets. As such, it has applications not only within combinatorial geometry, but also in optimization and property testing.

We discuss related concepts as Helly-type theorems and fractional Helly-type theorems. Apart from that, we focus on the following question: What conditions we need to put on a family of n sets in the plane where no k+1 sets intersect, in order to conclude that the number of intersecting k-tuples is at most cn^{k-1} for some constant c?

We provide a sufficient topological condition which includes much more families than convex sets, for which the answer was known.

3.11 Metric Violation Distance

Benjamin Raichel (University of Texas – Dallas, US)

Joint work of Chenglin Fan, Anna Gilbert, Benjamin Raichel, Rishi Sonthalia, Gregory Van Buskirk
Main reference Chenglin Fan, Benjamin Raichel, Gregory Van Buskirk: "Metric Violation Distance: Hardness and Approximation", in Proc. of the Twenty-Ninth Annual ACM-SIAM Symposium on Discrete Algorithms, SODA 2018, New Orleans, LA, USA, January 7-10, 2018, pp. 196–209, SIAM, 2018.
URL https://doi.org/10.1137/1.9781611975031.14

We introduce and study the metric violation distance problem: given a set of pairwise distances, represented as graph, modify the minimum number of distances such that the resulting set forms a metric. Three variants are considered, based on whether distances are allowed to only decrease, only increase, or the general case which allows both decreases and increases.

We show that while the decrease only variant is polynomial time solvable, the increase only and general variants are Multicut hard. By proving interesting necessary and sufficient conditions on the optimal solution, we provide approximation algorithms approaching our hardness bounds.

3.12 Hitting Convex Sets with Points

Natan Rubin (Ben Gurion University – Beer Sheva, IL)

Main reference Natan Rubin: "An Improved Bound for Weak Epsilon-Nets in the Plane", in Proc. of the 59th IEEE Annual Symposium on Foundations of Computer Science, FOCS 2018, Paris, France, October 7-9, 2018, pp. 224–235, IEEE Computer Society, 2018.
URL https://doi.org/10.1109/FOCS.2018.00030

We show that for any finite set P of points in the plane and $\epsilon > 0$ there exist roughly $\epsilon^{-3/2}$ points that pierce every convex set K with that encompasses at least an ϵ-fraction of P. This is the first improvement of the bound of $O(\epsilon^{-2})$ that was obtained in 1992 by Alon, Bárány, Füredi and Kleitman for general point sets in the plane.

3.13 Hamiltonicity for convex shape Delaunay and Gabriel graphs

Maria Saumell (The Czech Academy of Sciences – Prague, CZ & Czech Technical University – Prague, CZ)

Joint work of Maria Saumell, Prosenjit Bose, Pilar Cano, Rodrigo I. Silveira
Main reference Prosenjit Bose, Pilar Cano, Maria Saumell, Rodrigo I. Silveira: "Hamiltonicity for Convex Shape Delaunay and Gabriel Graphs", in Proc. of the Algorithms and Data Structures – 16th International Symposium, WADS 2019, Edmonton, AB, Canada, August 5-7, 2019, Proceedings, Lecture Notes in Computer Science, Vol. 11646, pp. 196–210, Springer, 2019.
URL https://doi.org/10.1007/978-3-030-24766-9_15

We study Hamiltonicity for some of the most general variants of Delaunay and Gabriel graphs. Instead of defining these proximity graphs using circles, we use an arbitrary convex shape $\mathcal{C}$. Let S be a point set in the plane. The k-order Delaunay graph of S, denoted k-$DG_{\mathcal{C}}(S)$, has vertex set S and edge pq provided that there exists some homothet of $\mathcal{C}$ with p and q on its boundary and containing at most k points of S different from p and q. The k-order Gabriel graph k-$GG_{\mathcal{C}}(S)$ is defined analogously, except for the fact that the homothets considered are restricted to be smallest homothets of $\mathcal{C}$ with p and q on its boundary.

We provide upper bounds on the minimum value of k for which k-$GG_{\mathcal{C}}(S)$ and k-$DG_{\mathcal{C}}(S)$ are Hamiltonian. In particular, we give upper bounds of 24 for every $\mathcal{C}$ and 15 for every point-symmetric $\mathcal{C}$. We also improve the bound for even-sided regular polygons. These constitute the first general results on Hamiltonicity for convex shape Delaunay and Gabriel graphs.

3.14 The maximum level vertex in an arrangement of lines

Micha Sharir (Tel Aviv University, IL)

Joint work of Dan Halperin, Sariel Har-Peled, Eunjin Oh, Kurt Mehlhorn

The level of a point p in an arrangement of a set L of n lines is the number of lines that lie strictly below p. The problem is to find a vertex of maximum level. It was posed as Exercise 8.13 in the "Dutch" textbook, but it hides much more than meets the eye when L is not in general position. We present structural properties of maximum-level vertices (in degenerate arrangements) and develop algorithms that find such a vertex in near-linear time.

3.15 The Blessing of dimensionality: when higher dimensions help

Hans Raj Tiwary (Charles University – Prague, CZ)

Problems in CG often suffer from a curse of dimensionality in that typical dependence of algorithms is exponential in the dimension. In Linear Programming however one can often drastically reduce the size of an LP by introducing extra variables. In geometric terms, many interesting polytopes have exponentially many vertices and facets but are projections of polytopes that can have polynomially many facets. I will present a communication game to obtain such size reductions and illustrate it with a (non-geometric) example: Spanning trees.

3.16 Competitive Searching for a Line on a Line Arrangement

Marc van Kreveld (Utrecht University, NL)

Joint work of Quirijn Bouts, Thom Castermans, Arthur van Goethem, Marc van Kreveld, Wouter Meulemans
Main reference Quirijn Bouts, Thom Castermans, Arthur van Goethem, Marc van Kreveld, Wouter Meulemans: "Competitive Searching for a Line on a Line Arrangement", in Proc. of the 29th International Symposium on Algorithms and Computation, ISAAC 2018, December 16-19, 2018, Jiaoxi, Yilan, Taiwan, LIPIcs, Vol. 123, pp. 49:1–49:12, Schloss Dagstuhl – Leibniz-Zentrum fuer Informatik, 2018.
URL http://dx.doi.org/10.4230/LIPIcs.ISAAC.2018.49

We discuss the problem of searching for an unknown line on a known or unknown line arrangement by a searcher S, and show that a search strategy exists that finds the line competitively, that is, with detour factor at most a constant when compared to the situation where S has all knowledge. In the case where S knows all lines but not which one is sought, the strategy is 79-competitive. We also show that it may be necessary to travel on Omega(n) lines to realize a constant competitive ratio. In the case where initially, S does not know any line, but learns about the ones it encounters during the search, we give a 414.2-competitive search strategy.

3.17 Stability analysis of shape descriptors

Kevin Verbeek (TU Eindhoven, NL)

Joint work of Wouter Meulemans, Kevin Verbeek, Jules Wulms
Main reference Wouter Meulemans, Kevin Verbeek, Jules Wulms: "Stability analysis of kinetic orientation-based shape descriptors", CoRR, Vol. abs/1903.11445, 2019.
URL https://arxiv.org/abs/1903.11445

Motivated by the analysis and visualization of moving points, we study orientation-based shape descriptors on a set of continuously moving points, specifically the minimum oriented bounding box. The optimal orientation of this box may be very unstable as the points are moving, which is undesirable in many practical scenarios. If we bound the speed with which the orientation of the box may change, this may increase the area. In this talk we study the trade-off between stability and quality of oriented bounding boxes.

We first show that there is no stateless algorithm, an algorithm that keeps no state over time, that both approximates the minimum area of the oriented bounding box and achieves continuous motion. On the other hand, if we can use the previous state of the shape descriptor to compute the new state, then we can define "chasing" algorithms that attempt to follow the optimal orientation with bounded speed. Under mild conditions, we show that chasing algorithms with sufficient bounded speed approximate the minimum area at all times.

3.18 A Motion Planning Algorithm for the Invalid Initial State Disassembly Problem

Nicola Wolpert (University of Applied Sciences – Stuttgart, DE)

Joint work of Daniel Schneider, Elmar Schömer, Nicola Wolpert
Main reference Daniel Schneider, Elmar Schömer, Nicola Wolpert: "A motion planning algorithm for the invalid initial state disassembly problem", in Proc. of the 20th International Conference on Methods and Models in Automation and Robotics, MMAR 2015, Międzyzdroje, Poland, August 24-27, 2015, pp. 35–40, IEEE, 2015.
URL https://doi.org/10.1109/MMAR.2015.7283702

Sampling-based motion planners are able to plan disassembly paths at high performance. They are limited by the fact that the input triangle sets of the static and dynamic object need to be free of collision in the initial and all following states. In real world applications, like the disassembly planning in car industry, this often does not hold true. Beside data inaccuracy, this is mainly caused by the modeling of flexible parts as rigid bodies, especially fixture elements like clips. They cause the invalid initial state disassembly problem. In the literature there exists no algorithm that is able to calculate a reasonable disassembly path for an invalid initial state. Our novel algorithm overcomes this limitation by computing information about the flexible parts of the dynamic object and incorporating this information into the disassembly planning.

4 Open problems

4.1 Problem 1

Jeff Erickson (University of Illinois – Urbana-Champaign, US)

Given a directed graph G embedded on a surface S of genus 2 with some marked edges, compute whether there exists a closed walk in G with more marked edges than unmarked edges that is contractible on S. What is the running time for solving this problem? Is this problem even decidable?

A bit of context to the problem:

This is the simplest open special case of finding negative-weight contractible walks in weighted directed graphs. Even the more general problem can be solved in polynomial time for directed graphs on the torus (via homology and linear programming) or directed graphs on surfaces with boundary (via CFG-shortest-paths algorithms). More generally, negative-weight walks with trivial (integer) homology on *any* surface can be found in polynomial time.

An affirmative answer to the following question would yield an algorithm for this problem: Is there a function $f(n)$, such that for any n-vertex directed graph with some edges marked, the shortest majority-marked contractible walk has at most $f(n)$ edges? In particular, a polynomial bound on $f(n)$ would imply a polynomial-time algorithm.

There is no such upper bound for negative contractible walks in real-weighted graphs. Even in graphs with constant complexity on the torus, the shortest negative contractible walk can be arbitrarily long. Any hardness (or undecidability) results for real-weighted graphs and/or higher-genus surfaces would also be interesting.

4.2 Problem 2

Peyman Afshani (Aarhus University, DK)

Consider the following two problems.

1. This is a weighted version of the level set problem. Consider a set P consisting of n points in the plane. We say a subset $S \subset P$ is *separable* if S can be separated from $P \setminus S$ using a line. Consider a function $f : P \to \mathbb{R}^+$. Given a value $w \in \mathbb{R}$, a separable subset $S \subseteq P$ is a w-set if $f(S) \leq w$ and for all separable sets S' such that $S \subsetneq S'$ we have $f(S') > w$. Find a non-trivial upper/lower bound on the maximum number of w-sets for given w. In particular, can we have $\Omega(n^2)$ w-sets for some w? Can we prove an upper bound of $O(n^{3/2})$ for any w?
2. Given a set of n points in the plane with real-valued weight, compute a centerpoint of P, preferably in $O(n \log n)$ time.

4.3 Problem 3

Guillaume Moroz (INRIA Nancy – Grand Est, FR)

Given pairwise distinct areas of all four sides A_1, A_2, A_3, A_4, the volume V, and the radius of the enclosing ball R, how many tetrahedra with this property exist up to isometry?

Conjecture: There exist at most 6 for any combination of those properties.

4.4 Problem 4

Stefan Langerman (UL – Brussels, BE)

Given a set of points P and their Delaunay triangulation, find a vertex separator of size $n^{\alpha}, \alpha < 1$ in $o(n \log n)$ time for this graph. Similarly, in three dimensions, find a separator with those properties of the graph of the convex hull.

4.5 Problem 5

Antoine Vigneron (Ulsan National Institute of Science and Technology, KR)

Given a partition of the plane, directional cones for each component of the partition, and two points s and t. We want to compute whether there exists a trajectory from s to t which respects the directional constraints. For single component directional cones there is a $O(n \log n)$ algorithm known. For multiple components it is NP-hard. What about symmetric cones with two components? *Clarification*: If the directional cones have multiple connected components, then we are not allowed to use directions from different components during the same visit of the area.

4.6 Problem 6

Hans Raj Tiwary (Charles University – Prague, CZ)

Do there exist polytopes P_1, P_2, Q such that
1. $P_1 \times P_2$ is a projection of Q, and
2. $xc(Q) \leq xc(P_1) + xc(P_2) - -1$,

where $xc(P)$ of a polytope P denotes the minimum number of facets of any polytope that projects to P. It is known that if either P_1 or P_2 is a pyramid, then this does not exist.

4.7 Problem 7

Birgit Vogtenhuber (TU Graz, AT)

Given a straight-line drawing of a complete graph K_n and a value $k \in \mathbb{N}$, does there exist a 2-coloring of edges such that there are less than k monochromatic crossings? Does this problem have a polynomial time algorithm?

4.8 Problem 8

Hsien-Chih Chang (Duke University – Durham, US)

Given two strings A and B ($n := |A|, m := |B|$), does there exist a data structure with
- $O((nm)^{1-\epsilon})$ preprocessing time (for some $\epsilon > 0$)
- $\tilde{O}(m^{1-\delta})$ query time for some $\delta > 0$ checking whether $LCS(A[i \ldots j], B) = LCS(A[i \ldots j+1], B)$

This problem is interesting for any values of m. Especially, data structure for $m = n^c$ for some $c > 0$ would be sufficient. Note that computing $LCS(A[i \ldots j], B)$ is equivalent to computing the shortest path distance from node i at the top row to node j at the bottom row in the dynamic-programming graph, which is planar.

4.9 Problem 9

Hans Raj Tiwary (Charles University – Prague, CZ)

Given 0/1 polytopes P_1, P_2, Q (i.e., polytopes where all vertices are in $\{0,1\}^d$) defined by inequalities, we want to know whether $P_1 + P_2 = Q$, where the addition is the Minkowski sum. For general polytopes this problem is known to be (co)NP-complete.

4.10 Problem 10

Suresh Venkatasubramanian (University of Utah – Salt Lake City, US)

Consider the n-dimensional hypercube, and given n curves $\alpha_i : [0,1] \to [0,1]^n$ with
- $\alpha_i(0) = (0, \ldots, 0, 1, 0, \ldots, 0)$, where the ith position is 1
- $\alpha_i(1) = (1, \ldots, 1)$
- $\forall i, j\ \forall t \leq t', d(\alpha_i(t), \alpha_j(t)) \geq d(\alpha_i(t'), \alpha_j(t'))$

where d is the euclidean distance.

Question: What tools are applicable to this setting?

4.11 Problem 11

Maarten Löffler (Utrecht University, NL)

Let T be a tree with $n = k^2$ vertices. A perfect plane grid drawing of a tree is a bijection from the tree nodes to the nodes of a regular $k \times k$ grid such that

1. the edges are preserved and embedded by straight lines, and
2. there are no crossing edges, and
3. no edge is going through a node.

Question 1: What is the runtime of deciding whether this is possible for a tree. (NP-hard? polynomial?)

Let d be the maximal degree of a node in T.

Question 2: What is a function of the maximal degree $d(n)$ such that a perfect plane grid drawing is always possible?

Question 3: Even for $d = 3$, is such an embedding always possible?

4.12 Problem 12

Kyle Jordan Fox (University of Texas – Dallas, US)

Given the complete graph K_n with each edge colored either red or blue, and each edge has a non-negative edge weight. Find the minimum weight perfect matching with an odd number of red edges.

Known: There exists a randomized pseudo-polynomial algorithm.

Question: What is the complexity of this problem?

Applications: Max cut on surface graphs, matroid girth.

4.13 Problem 13

Siu-Wing Cheng (HKUST – Kowloon, HK)

This open problem is about self-improving sorting. Given numbers $x_1, \ldots, x_n$ drawn independently from distributions $D_1, \ldots, D_n$, i.e. $x_i \sim D_i$ for all i. We first allow for an arbitrarily long learning phase which has $O(n^{1+\epsilon})$ space. Then there is the limiting phase in which we want to sort new instances in $O(\frac{1}{\epsilon}(n + H))$ expected time with high probability, where H is the sum of the entropies of the D_i. There was a result at ISAAC'18 about this problem. Can more general input model be allowed?

4.14 Problem 14

Maria Saumell (The Czech Academy of Sciences – Prague, CZ & Czech Technical University – Prague, CZ)

Shamos [1] conjectured that the Delaunay triangulation always contains a Hamiltonian cycle. Dillencourt [2] disproved this conjecture, but he also showed that Delaunay triangulations are *almost* Hamiltonian [3], in the sense that they are 1-tough.[1]

[1] A graph is *1-tough* if removing k vertices from it results in $\leq k$ connected components.

Given a planar point set S and two points $p, q \in S$, the k-Delaunay graph (k-DG) with vertex set S has an edge pq provided that there exists a disk with p and q on the boundary containing at most k points of S different from p and q. The following question arises: What is the minimum value of k such that the k-Delaunay graph of any point set S is Hamiltonian? Chang et al. [4] showed that 19-DG is Hamiltonian, and Abellanas et al. [5] lowered this bound to 15-DG. Currently, the lowest known bound is by Kaiser et al. [6] who showed that 10-DG is Hamiltonian. Despite this, it is conjectured that 1-DG is Hamiltonian [5]. Is this conjecture true?

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