Introduction to Statistical Inference

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What is Statistics?

- Relatively new discipline
- Scientific revolution in the 20th century
- Data and computing revolutions in the 21st century
- The world is stochastic rather than deterministic
- Probability theory used to model stochastic events

- Statistical inference: Learning about what we do not observe (parameters) using what we observe (data)
- Without statistics: wild guess
- With statistics: **principled guess**
  1. assumptions
  2. formal properties
  3. measure of uncertainty

Statistics for Social Scientists

- Quantitative social science research:
  1. Finding a substantive question
  2. Constructing theory and hypothesis
  3. Designing an empirical study
  4. Using statistics to analyze data and test hypothesis
  5. Reporting the results

- No study in social sciences is perfect
- Statistical methods are no substitute for a good design
- Data analysis $=$ statistical theory (objective) $+$ judgement calls (subjective)
- Justifying your choices and increasing credibility
- Understanding and evaluating the assumptions of statistical methods

How Not to Study Statistics

- Learning statistics (or math in general) is different from learning substantive topics in political science

- “Skip equations, and try to understand them from the context...”
- “I should be able to get the intuition without knowing the math...”
- “Move on, and try to figure out the next topic...”
- “Do you understand so far? Uhhh... Yes.”
- “I put all of the formulae on flashcards to study for the exam...”
- “I am just going to copy down the formula...”
- “What formula am I supposed to use here?”

- Goal of the course: become a sophisticated user of statistical methods and build a solid foundation for future study

Three Modes of Statistical Inference

1. Descriptive Inference: summarizing and exploring data
   - Inferring "ideal points" from rollcall votes
   - Inferring "topics" from texts and speeches
   - Inferring "social networks" from surveys
2. Predictive Inference: forecasting out-of-sample data points
   - Inferring future state failures from past failures
   - Inferring population average turnout from a sample of voters
   - Inferring individual level behavior from aggregate data
3. Causal Inference: predicting counterfactuals
   - Inferring the effects of ethnic minority rule on civil war onset
   - Inferring *why* incumbency status affects election outcomes
   - Inferring whether the lack of war among democracies can be attributed to regime types

What is Causal Inference?

- Comparison between factual and counterfactual
- Incumbency effect
- What would have been the election outcome if a candidate were not an incumbent?
- Resource curse thesis
- What would have been the GDP growth rate without oil?
- Democratic peace theory
- Would two autocracies have escalated crisis in the same situation?
- SUPPLEMENTARY READING: Holland, P. (1986). Statistics and causal inference. (with discussions) *Journal of the American Statistical Association*, Vol. 81: 945–960.

Defining Causal Effects

- Units: $i = 1, \ldots, n$
- "Treatment": $T_i = 1$ if treated, $T_i = 0$ otherwise
- *Observed* outcome: Y_i
- Pre-treatment covariates: X_i
- Potential outcomes: $Y_i(1)$ and $Y_i(0)$ where $Y_i = Y_i(T_i)$

Voters	Contact	Turnout		Age	Party ID
i	T_i	$Y_i(1)$	$Y_i(0)$	X_i	X_i
1	1	1	?	20	D
2	0	?	0	55	R
3	0	?	1	40	R
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	1	0	?	62	D

- Causal effect: $Y_i(1) - Y_i(0)$

The Key Assumption

- No interference between units:

$$Y_i(T_1, T_2, \ldots, T_n) = Y_i(T_i)$$

- Stable Unit Treatment Value Assumption (SUTVA)
- Potential violations: spill-over effects, contagion, cluster randomized experiments
- Potential outcomes are thought to be fixed for each individual
- J-valued treatment $\longrightarrow$ J potential outcomes for each unit

Causal Effects of Immutable Characteristics

- "No causation without manipulation" (Holland, 1986)
- Immutable characteristics; gender, race, age, etc.
- What does the causal effect of gender mean?

- Causal effect of having a female politician on policy outcomes (Chattopadhyay and Duflo, 2004 *QJE*)
- Causal effect of having a discussion leader with certain preferences on deliberation outcomes (Humphreyes *et al.* 2006 *WP*)
- Causal effect of a job applicant's gender/race on call-back rates (Bertrand and Mullainathan, 2004 *AER*)

Average Causal Effects

- Sample Average Causal Effect (SACE):

$$\frac{1}{n}\sum_{i=1}^{n} Y_i(1) - Y_i(0)$$

- Population Average Causal Effect (PACE):

$$\mathbb{E}(Y_i(1) - Y_i(0))$$

- Population Average Treatment Effect for the Treated (PATT):

$$\mathbb{E}(Y_i(1) - Y_i(0) \mid T_i = 1)$$

- Causal heterogeneity: Zero ACE doesn't mean zero effect for everyone
- Other quantities: Conditional ACE, Quantile Causal Effects, etc.

Design Considerations

- No causation without manipulation (Holland)
- Randomization of the treatment enables the unbiased estimation of average causal effects
  - Laboratory experiments
  - Survey experiments
  - Field experiments
- Natural experiments with haphazard treatment assignment
  - Birthdays
  - Weather
  - Close elections
  - Arbitrary administrative rules
- Observational studies with statistical control
  - Regressions
  - Matching
- Tradeoff between internal and external validity

Truncation by Death

- Units: patients (students)
- Treatment: a new medicine (a new teaching program)
- Outcome: blood pressure (the end-of-year test score)
- Truncation: some patients die (some students drop out)
- Blood pressure (test score) undefined for the dead (drop-outs)!

- Randomized evaluation of Mexican health care
- Units: individuals
- Treatment: universal health insurance
- Outcome: satisfaction with the received health care
- Truncation: some have not been to clinics or hospitals

- SUPPLEMENTAL READING: Rubin, D. B. (2006). Causal Inference Through Potential Outcomes and Principal Stratification: Application to Studies with “Censoring” Due to Death *Statistical Science*, Vol. 21: 299 – 309. Skip Sections 4 – 6.

Principal Stratification

- Comparison between the dead and the alive is invalid
  - Potential selection bias
  - Causal effects need to be defined at the unit level

- Binary treatment: $T_i \in \{0, 1\}$
- Potential truncation variable: $W_i(1), W_i(0)$
- Observed truncation variable: $W_i = W_i(T_i)$
- Potential outcomes: $Y_i(0, 0), Y_i(1, 0)$
- $Y_i(0, 1), Y_i(1, 1)$ do not exist
- Observed outcome: $Y_i = Y_i(T_i, W_i)$ for $W_i = 0$

- Causal effect: $Y_i(1, 0) - Y_i(0, 0)$ for "always-survivors"
- Not defined for other types (principal strata)

Causal Effects vs. Causal Mechanisms

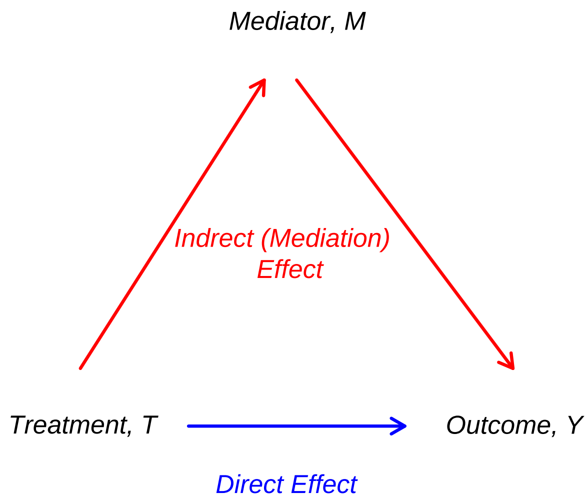
- Causal inference is a central goal of social science research
- Randomized experiments are seen as **gold standard**
- But, experiments are a **black box**
- Can only tell *whether* the treatment causally affects the outcome
- Not *how* and *why* the treatment affects the outcome
- Qualitative research uses process tracing

- How can quantitative research be used to identify causal mechanisms?

- SUPPLMENTAL READING: Imai, K., Tingley, D., and Yamamoto, T. (2009). Experimental Identification of Causal Mechanisms. Available at http://imai.princeton.edu/research/Design.html Read only Section 2.

Direct and Indirect Effects

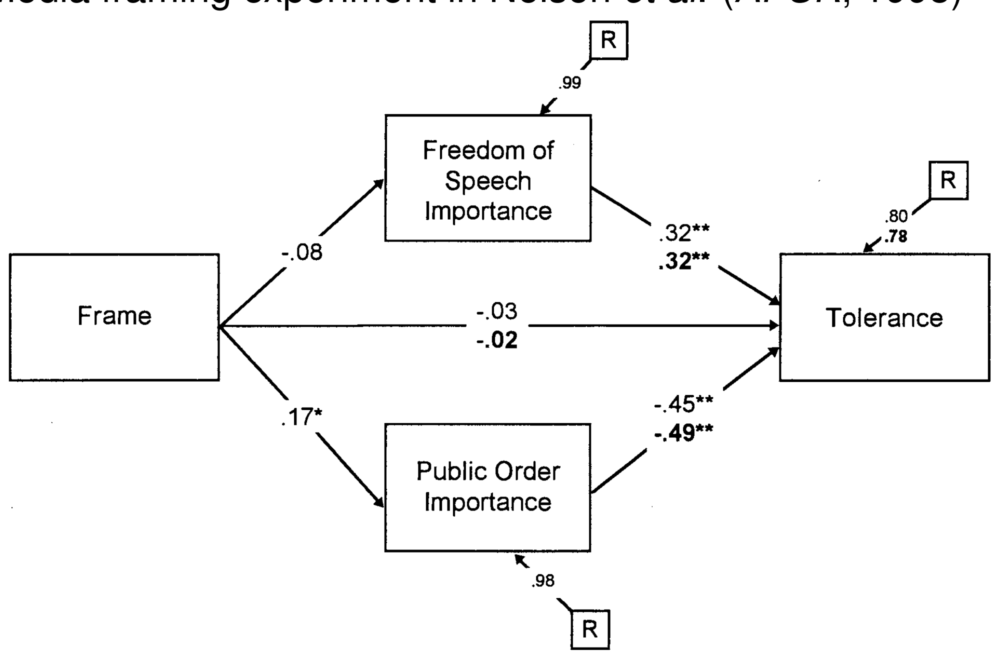
- Causal mediation analysis:



- Direct and indirect effects; intermediate and intervening variables

Causal Mediation Analysis in American Politics

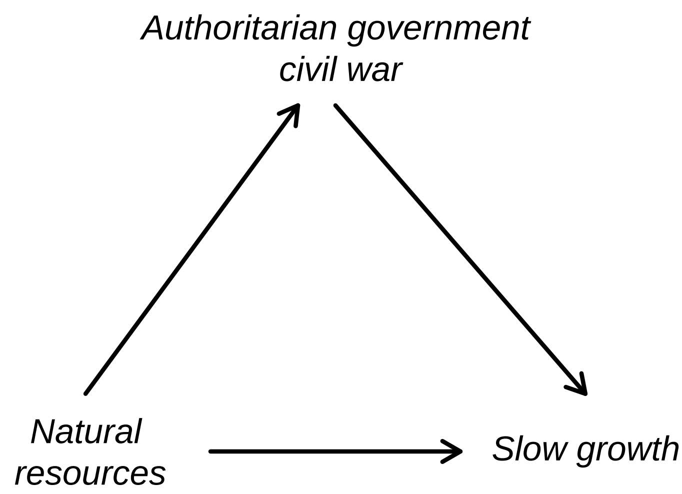
- Media framing experiment in Nelson *et al.* (*APSR*, 1998)



- Path analysis, structural equation modeling

Causal Mediation Analysis in Comparative Politics

- Resource curse thesis



- Causes of civil war: Fearon and Laitin (*APSR*, 2003)

Causal Mediation Analysis in International Relations

- The literature on international regimes and institutions
- Krasner (*International Organization*, 1982)

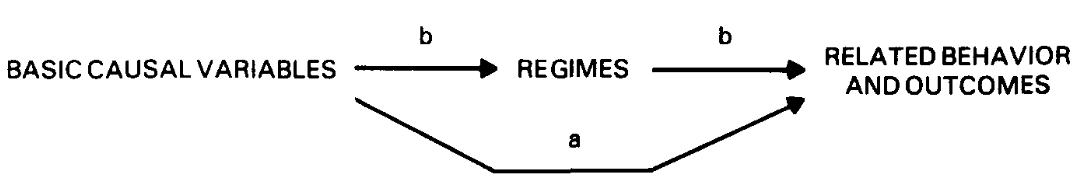


Figure 2

- Power and interests are mediated by regimes

Potential Outcomes Notation for Causal Mechanisms

- Treatment: $T_i \in \{0, 1\}$
- Potential mediators: $M_i(t)$ where $M_i = M_i(T_i)$
- Potential outcomes: $Y_i(t, m)$ where $Y_i = Y_i(T_i, M_i(T_i))$
- Total causal effect: $\tau_i \equiv Y_i(1, M_i(1)) - Y_i(0, M_i(0))$

- Indirect effects:

$$\delta_i(t) \equiv Y_i(t, M_i(1)) - Y_i(t, M_i(0))$$

- Change the mediator from $M_i(0)$ to $M_i(1)$ while holding the treatment constant at t
- Indirect effect of the treatment on the outcome through the mediator under treatment status t
- $Y_i(t, M_i(t))$ is observable but $Y_i(t, M_i(1-t))$ is not

- Direct effects:

$$\zeta_i(t) \equiv Y_i(1, M_i(t)) - Y_i(0, M_i(t))$$

- Change the treatment from 0 to 1 while holding the mediator constant at $M_i(t)$

- Total effect = mediation (indirect) effect + direct effect:

$$\tau_i = \delta_i(t) + \zeta_i(1-t) = \frac{1}{2}\sum_{t=0}^{1} \delta_i(t) + \zeta_i(t)$$

- Quantities of interest: Average Causal Mediation Effects,

$$\mathbb{E}(\delta_i(t)) = \mathbb{E}\{Y_i(t, M_i(1)) - Y_i(t, M_i(0))\}$$

Concluding Remarks

- Descriptive inference: becoming increasingly important in the age of data revolution
- Predictive inference: under-utilized in social sciences but could be used more for theory testing and policy making
- Causal inference: most difficult but most casually used
  - Potential outcomes framework, dating back to Neyman (1923)
  - Importance of design considerations
  - Credibility of causal assumptions